Can Access Price Indexation Promote Efficient Investment in Next Generation Networks?*

David Henriques†

Ofcom – UK Regulator and Competition Authority

This version: September 26, 2012

Abstract

The existing literature on access price and investment has pointed out that networks underinvest under a regime of mandatory access provision with a fixed access price per end-user. In this paper we propose a new access pricing rule, the indexation approach, i.e., the access price, per end-user, that network i pays to network j is function of the investment levels set by both networks. We show that the indexation can enhance economic efficiency beyond what is achieved with a fixed access price. In particular, the access price indexation can simultaneously induce lower retail prices and higher investment and social welfare as compared to both the fixed access pricing and the regulatory holidays. Furthermore, we show that the indexation can implement the socially efficient investment, which would be impossible to obtain under a fixed access pricing. Our results contradict the notion that investment efficiency must be sacrificed for gains in pricing efficiency.

Keywords: Access pricing, Internet, Investments, Next Generation Networks, Regulation.

JEL Classification: L43, L51, L96.

* I thank Steffen Hoernig for extensive comments, suggestions and discussions on matters related to this paper. I also wish to thank Bruno Basalisco, David Sauer, Diogo Lucena, Fabio Manenti, Giovanni Tabacco, Heski Bar-Isaac, Inês Morão, Luís Cabral, Paula Gonzalez, Pedro P. Barros, Tiago Pires, as well seminar participants at the Informal Research Workshop at Nova SBE, the Micro Lunch Seminar at NYU–Stern, the 5th Annual Meeting of the Portuguese Economic Journal (University of Aveiro), the XXVI Jornadas de Economía Industrial (University of Valencia), the Conference on the Economics of Information and Communication Technologies (TELECOM ParisTech), the V Intertic Conference on Innovation, Competition and the New Economy (University of Venice, Ca' Foscari), the ASSET 2011 Conference (University of Évora), the UECE Lisbon Meetings 2011 in Game Theory and Applications, the Royal Economic Society 7th PhD Presentation Meeting (Queen Mary University of London), the ICT workshop 2012 (FEP, Porto), the ICTNET 4th workshop (Imperial College, London), the IESE Business School (Barcelona), CRA (Oakland CA), Bates White (Washington DC) and Ofcom (Economists Meeting) for useful comments and suggestions on earlier drafts. All errors are my own. I gratefully acknowledge the PLUG 2008 Prize in Telecommunications (1st honorable mention) sponsored by Apritel – Telecommunications Operators Association.

† Contact: dthenriques@gmail.com. Home-page: `http://dthenriques.orgfree.com`.

1 Introduction

Motivation. A key concern for the United States and Europe is the timely rollout of Next Generation Networks (NGNs).[1] Optical fibre is at the core of NGNs and is considered the future of telecommunications infrastructure, since it allows faster and wider transmission of all sorts of information than copper-based networks.[2] Significant investments are required to supply the necessary communications infrastructure that consumers and firms demand in order to effectively compete in nowadays' knowledge based society. While the technology exists today, it is uncertain when and to what extent it will be deployed by network operators. In 2009, fibre to the home (FTTH) had reached nearly 13% penetration of U.S. households in terms of homes passed and 4% in terms of homes connected (RVA LLC Market Research & Consulting, April 2009), while in Germany and Spain FTTH covered less than 5% of the households, and in Italy less than 10% (IDATE Consulting & Research, February 2010). At the end of 2010 the percentage of subscribers out of total homes passed by fibre was 17.5% in Europe, 34% in the United States and 39% in Japan (IDATE Consulting & Research, 9 February 2011).[3] These facts suggest that residential and business users, namely in Europe, are unsure about the benefits of FTTH given the level of retail prices charged by the operators.

Telecommunications regulators have the task of encouraging investment and innovation and simultaneously ensuring that networks remain competitive, as competition is a vital matter for end-users and for businesses relying on the new networks. However, regulators seem to face a tradeoff between static and dynamic efficiency. On the one hand, static regulation reduces the extent to which operators exert market power on the downstream market, inducing retail prices to converge closer to marginal cost.[4] On the other hand, a fixed access price based on cost, while it may promote the statically efficient use of the network, discourages investment (dynamic inefficiency) since the returns that can be earned by investors are constrained by the access price set by the regulator.[5]

[1] The idea behind the NGN is that a single network infrastructure transports all information and services (e.g. voice, data, high definition TV, interactive gaming) allowing to increase transmission speeds by encapsulating information into packets.

[2] Aside from fibre, there are a number of alternative technologies capable of supporting NGN access such as: coaxial cable, mobile and fixed wireless networks. Since fibre is one of the fastest technologies for content transmission (both downloading and uploading), debates on wired NGN access have focused on fibre deployment.

[3] Asian carriers occupy eight out of the top 10 spots in terms of fibre subscriber numbers. Japan is ranked number 1 with 13,839,000 subscribers. None of the top 10 FTTH market players is from Europe. Asian operators were the first to strongly invest in fibre rollout, and have in 2011 achieved virtually complete coverage in their respective national markets.

[4] Setting access conditions in network industries is an essential issue for regulators to avoid anti-competitive behavior on the part of the networks (bottleneck-facility owners). In particular, access regulation is important to avoid that networks deter entry by refusing access to competitors and to provide competitors with reasonable access prices, guaranteeing competitive parity among operators.

[5] Imposing open access with a fixed access price also calls to mind the classical free-riding problem in static frameworks (see Olson (1965), Chamberlin (1974) and McGuire (1974)). The literature on free-riding points out that the investment level of equilibrium in public goods is lower than the Pareto efficient investment level. In a monopolistic market structure the free-riding problem vanishes; however, the retail price would become inflated, generating potential welfare losses.

The question under the most used costing methodologies[6] is: How much price efficiency must be sacrificed to achieve a desired level of investments? In this paper we question whether such a tradeoff always exists. The main challenge is thus to create an access price rule that responds to the question: How to encourage investment in network (bottleneck) infrastructures without lessening downstream price competition relatively to the fixed access pricing methodology?

Description of the paper. We consider a context of bilateral one-way access, i.e., there are two bottleneck facilities (networks) forced to provide access to each other under some regulatory conditions.[7] Underinvestment derives from the inability of networks to capture the full social benefit from investment. The problem of access obligations mandated by regulation is that they diffuse the investment benefits among operators and consumers while the investment cost is concentrated on the investor (network owner). Hence, underinvestment in infrastructure is aggravated by the non-exclusivity imposed by regulation together with the fact that investment is costly.

Let a denote the access price per subscriber under the fixed access price methodology, which is currently used by regulators. We compare the optimal fixed access price a^* to a new access price rule, which we call of access price indexation, in terms of retail prices, fibre coverage and social welfare levels. Under the indexation approach the access price is defined by the regulator as a function of the operators' investments in fibre coverage.

The main purpose of the new access price proposal is to reward or punish operators depending on the investments made by each one. On the one hand, the indexation rule should reward an operator i for covering cities by fibre, lowering the access price a_j charged by j when i needs j's network to serve subscribers. Thus, the indexation rule can grant a competitive advantage at the retail price level to whom invests relatively more. On the other hand, the new rule intends to punish the operator that invests relatively less by increasing its access price to the competing network. The indexation can impose a competitive disadvantage to free riders and operators that invest less when competing in the downstream market. This is also a solution to investors internalize the positive spillovers exerted from their investments in the sense that a_i should increase in I_i. With the access price indexation conveniently chosen by the regulator, the "dilemma" faced by the networks is that, whatever the other does and as long profits are non-negative, each network is better off investing relatively more since investments are a source of a competitive advantage in retail prices. For example, by using a simple linear access pricing rule depending on investments (I_i, I_j) by operator i and j, $a_i(I_i, I_j) = xI_i - yI_j$,

[6] In 2009, the long-run-incremental cost was the costing approach most often applied to European markets for wholesale access at a fixed location (64%) and the second most used for wholesale broadband access (46%). The fully distributed cost approach had a share of 32% and 54%, respectively. With respect to investment in NGN, the Commission recommendation (European Commission, 20 September 2010) suggests a risk premium when setting access prices to the unbundled fibre loop in order to compensate the investor for bearing the risk of failure alone.

[7] This is different from two-way access since in this model end-users do not interact with each other, whilst two-way access environments are characterized by end-user interconnection.

where $(x, y) \in \mathbb{R}_+^2$ are regulatory parameters, we can create a causal link from retail price competition to (investments in) fibre coverage. This rule, seemingly consistent with current law, pushes networks to compete harsher in investments.

We show that the new rule increases economic efficiency as compared to the fixed access price methodology. The indexation methodology dominates both the fixed access pricing rule and the regulatory holidays policy in terms of retail price efficiency (or equivalently, the number of consumers served), investment efficiency (i.e., the number of cities covered by fibre) and social welfare. Furthermore, we show that the indexation rule can promote the socially optimal (first-best) investment while that is unfeasible either with a fixed access price or with regulatory holidays.[8] The intuition for these results is the following. Since part of the benefits generated by investments is retained by consumers, the monopolistic (regulatory holidays) outcome is not only inefficient in retail price but also inefficient in investment. Under a fixed access price the introduction of competition in the downstream market can only deteriorate investment efficiency, while under the indexation rule networks have incentives to compete in investments as a means to gain a competitive advantage in the downstream market. Hence, by choosing the proper access price indexation, the regulator can encourage networks to invest until some level, which may be the socially optimal, as long networks have non-negative profits. Using the indexation rule both networks can have higher incentives to invest even if the equilibrium access price is the same than using the fixed access method. For example, in the linear access pricing case, the regulator can control the investments through the levels of x and y, and control the equilibrium access price through the difference $x - y$. Specifically, in the symmetric equilibrium case, $I_i = I_j$, $a_i = (x - y)\, I_i$ and suppose that x and y present relatively high values but the difference $x - y$ is relatively small. Then, the regulator is providing incentives to investment while keeping a relatively small access price. We conclude that market outcomes under the new rule lie outside the previously perceived "second-best efficiency frontier" under the fixed access price approach.

The main contribution of this paper is to show that an access price rule depending on investments, without being informationally more demanding, can improve economic efficiency both in terms of retail prices and investments as compared to a fixed access price rule. In a nutshell, the access price indexation is a feasible instrument that can enable a regulator to obtain higher social welfare.

Background. A crucial issue in the economics of regulation of NGN access is how to encourage operators to invest in infrastructure. Attempts to develop and invest in NGNs have been taken in many countries by National Regulatory Authorities (NRAs) and Governments. For example, in 2006, in Germany the incumbent operator Deutsche Telekom told the Government that would make these investments only if the Government granted regulatory holidays, i.e., the incumbent would be temporarily a monopolist without oblig-

[8] According to our model results, even assuming that the regulator knows with certainty all parameters, a fixed access price is condemned to be inefficient both in retail prices and investments.

ation to provide access to competitors at regulated prices.[9] In 2008, the Spanish NRA removed the requirement on Telefonica, the incumbent operator, to supply wholesale access service on FTTH. This verdict gave Telefonica a regulatory holiday on FTTH network access, similar to that held by Deutsche Telekom (ITU, 2009). The French model follows the cost-sharing perspective. It forces network operators, which may invest on their own, to make available access to ducts and supply information on planned civil works and fibre coverage, sharing the installation costs of additional fibre at other operators' request. Other options to stimulate the development on NGNs are the establishment of public-private partnerships (PPP), as has happened in Singapore and Australia, and the provision of credit lines and funds, for example in Portugal and, in a relatively small scale, in the United States.[10] In these cases Governments invest, provide funding or credit to kick-off the projects on NGNs and accelerate the fibre deployment.

The introduction of competition into historical monopolies in telecommunications has led to a number of research articles on access pricing issues, as regulators have been confronted with the need to set the rules on which operators should have access to each other's network. The vast majority of articles on access pricing assumes that access fees do not depend explicitly on investment levels.[11] Only recently some exceptions, as Hurkens and Jeon (2008), Nitsche and Wiethaus (2011), Klumpp and Su (2010) and Sauer (2011) have considered the idea of having access prices as function of strategic variables, namely, retail prices, quantities or investments.

Gans (2004) presented a model to study the impact of access price regulation on investment timing. In particular, Gans investigated if such regulation can improve investment timing on market outcomes, relatively to the social optimal, whilst encouraging price competition. First, it is shown that investment might be delayed vis-à-vis the socially efficient timing if one firm is "small". When two firms are "large", competition accelerates investment timing and the investment might be provided too rapidly at a cost higher than in the socially efficient solution. Second, the article shows that the regulator may use fixed access charges to induce the investment timing outcome to be socially efficient, by controlling the preemption incentives of other possible providers. Regulation may have thus an important role on preventing inefficient acceleration of facility investment.

[9] The European Commission, against the adoption of regulatory holidays, sent Germany a formal caution in February 2007, after repeated warnings that had been ignored. The case was taken to European Court (European Comission VS German Regulator and Deutsche Telekom) that ruled against German regulatory holidays in December 2009. See ITU (2009) and *EU court "sets precedent" in Germany telecoms ruling*, EurActiv, 4 December 2009, for further details.

[10] The Australian government decided in 2009 to invest and to be the majority shareholder of a $A43bn super-fast national broadband network. The U.S. government under President Barack Obama has allocated $USD7.2 billion to support broadband build-up. In Portugal, in 2009, a protocol on NGN was signed between the Government and four operators (Portugal Telecom, Sonaecom, Zon and Oni Communications), in which there is a commitment of all parties to invest in NGN. The Portuguese Government is committed to make available a credit line of, at least, EUR800 million.

[11] See Valletti (2003), Guthrie (2006) and Cambini and Jiang (2009) for excellent reviews on how access pricing and network investments have been investigated by the theoretical literature. This literature points to the need to consider more deeply the impact of access regulation on investments and on welfare.

De Bijl and Peitz (2004) explored situations of one-way access in which an integrated operator owns a network infrastructure and sells access directly to end-users and to a downstream operator. This article discusses the investment incentives of the integrated operator. In particular, De Bijl and Peitz show that it is possible to provide stronger incentives for the integrated operator to invest in infrastructure quality by increasing the sensitivity of the regulated access price on the network quality. Nonetheless, they do not consider any explicit form on how the access price should depend on quality.

Bourreau, Hombert et al. (2010) focused on industries in which an intermediate input (e.g. network access) is sold by vertically integrated firms that compete afterwards in prices with differentiated products in the downstream market with a non-integrated downstream firm. The article shows that upstream price competition with homogeneous inputs may not drive the input price down to marginal cost. The access price can be set at a level above marginal cost in order to lessen downstream competition between integrated and non-integrated firms. However, when final goods are strongly differentiated, downstream demands are practically independent among firms, and thus we are back to the classical Bertrand pricing result at the upstream level. The authors also derived conditions on the demand and cost functions under which an access price cap can repair the competitiveness in the upstream market.

Nitsche and Wiethaus (2011) analyzed the investment incentives and consumer welfare under different types of access regulation to NGNs. They show that for a given level of investment, risk-sharing (operators jointly deploy and share the costs of NGNs) induces the highest competitive intensity in the product market, followed by, respectively, long-run-incremental cost (LRIC), fully distributed costs (FDC) and regulatory holidays. They also show that, under uncertainty, FDC or regulatory holidays encourage highest investments, followed by, respectively, risk-sharing and LRIC. Moreover, according to simulation results, risk-sharing induces highest consumer surplus, since it puts together comparatively high ex-ante investment incentives with strong ex-post competitive intensity.

Hurkens and Jeon (2008), following a two-way access analysis with n network infrastructures, studied the retail benchmarking approach. They proposed access pricing rules that determine the access price as function of the retail prices charged by both networks. They show that such rule may induce the market outcome to achieve the socially efficient price at the retail level. Moreover, under two-part tariff competition, setting the access price paid by firm i to depend linearly on its average retail price and let networks invest in quality after the access pricing rule is determined and before they compete in two-part tariffs, it is possible to achieve both static and dynamic efficiency.

The closest independent research work to this is Sauer (2011) which compares, from the social perspective, the performance of different access regulatory regimes. Sauer's research focuses on (i) the regime of endogenous access charges per user, contingent on networks' investment levels and (ii) the regime of investment cost-sharing with lump-sum charges, i.e., the access price is proportional to investment costs of the competitor. Sauer shows that in the former it is possible to reach the socially efficient investment level without distorting downstream competition, whilst in the latter despite the higher investment level than with fixed access charges it is still below the socially efficient investment. Our paper is complementary since we focus on modelling techniques that differ at least in two major aspects. First, Sauer uses the Hotelling model with fully served consumers, while our model relies on the Hotelling model with hinterlands where consumers are fully served in the city center but may not be fully served in the hinterlands. Therefore, while in our model market power generates welfare effects, this does not happen in Sauer's model. Second, Sauer assumes that the access charge received by an operator is a non-negative function of its own investment. In this paper access prices depend on investments of both networks and may be negative.

Our paper is also related to the theory of yardstick competition and tournaments, and incentives in teams. See Lazear and Rosen (1981), Green and Stockey (1983), Holmström (1982), Nalebuff and Stiglitz (1983), and Shleifer (1985) for relevant theory developments. Under a context of uncertainty, an agent's low performance may be due to an unfavorable state of nature rather than to low effort. Such effects can be detected, to some extent, by comparing the agent's performance with that of other agents placed in similar conditions. The literature calls this scheme "yardstick competition". Marino and Zábojník (2001) show that a firm by organizing a tournament between two teams and transfer output from the team with inferior performance to the team with higher performance, this helps to solve (i) the free riding problem inside each team, and (ii) lessen the moral hazard problem. We use similar logic by creating a tournament between networks as a solution for an underinvestment problem in NGNs.

2 The Model

We start by presenting the basic modelling structure and providing the social optimum as benchmark case. Then we solve the model for different regulatory regimes: (i) a fixed access price, (ii) the access price indexation and (iii) regulatory holidays, and compare the outcomes in terms of fibre coverage, retail prices and welfare levels.

Consider the market for broadband access (FTTH) in which two networks labeled $i = 0, 1$ offer differentiated services. The timing of the model is summarized in Table 1. First, the regulator sets the rule for pricing access to bottleneck facilities.[12] Second, operators compete in investments (fibre coverage). In our framework this is the equivalent to have each operator choosing the number of cities to cover by fibre. Third, operators compete in retail prices in the downstream market in all cities covered by fibre. Investments are made only once but operators compete in the downstream market over many periods. Therefore, the third stage of the game may be considered as a reduced form of a dynamic

[12] Access pricing rules should be defined by the regulator as networks would otherwise choose too high access prices.

game of competition in the downstream market with a discounted stream of future profits. This structure of the game is natural as operators may decide on prices in the short-run and on investments in the long-run, while regulators decide on access prices in the very long-run.

Table 1: Timing of the model

I.	The regulator defines the access price rule per end-user, a_i, which operator i must follow when j is the accessing firm.
II.	Operators invest simultaneously and non-cooperatively in non-duplicable network infrastructure, which we interpret as NGNs infrastructure (FTTH).[13] Immediately after, operators observe the investment outcome.
III.	Operators compete simultaneously and non-cooperatively in retail prices.

Follows the description of each one of the participants in the model: the regulator, the networks, and the fibre subscribers (consumers in each city).

Regulator. The regulator can choose to fix the access price at some level $a_i = a^*$ or,[14] alternatively, to set an access price depending on operators' investment levels. For technical simplification, we assume a linear access price rule depending on investments defined by

$$a_i = xI_i - yI_j, \tag{1}$$

where $(x, y) \in \mathbb{R}^2$ are the regulatory parameters, and I_i, I_j denote the number of cities covered by fibre by operator i and j, respectively.[15] The total number of cities covered by fibre is denoted by I, where $I \equiv I_0 + I_1$. Since the investment level corresponds to the number of cities covered by fibre we assume that investments are perfectly observable by the regulator. For example, by observing the duct construction and networks' physical infrastructures for fibre optic deployment in cities.

We assume that the regulator is benevolent, i.e., maximizes social welfare, and can credibly commit ex-ante to impose an access price rule. Otherwise, networks would infer that once the investments had been made the regulator would set a new access price rule stimulating competition in retail prices. This would make networks less willing to invest than with a regulatory commitment on the access pricing rule.

[13] NGN access refers to the network segment connecting an end-user to the nearest location which houses the operator's equipment. In Europe, NGN access refers essentially to the introduction of fibre into the local loop.

[14] Regulatory holidays may be interpreted as the case when the regulator sets $a_i = \infty$ for a period of time.

[15] Under a linear indexation of access prices, the regulator will choose (x, y) such that in equilibrium networks' participation constraint binds, as shown in Lemma 1. Therefore, it is not possible to improve on investment efficiency without further distortions on retail prices. The linear indexation suits to show the main goal of this paper: access price indexation is better than fixed access prices regarding retail prices, investments and social welfare. We do not argue, though, that the linear indexation is the best functional indexation that a regulator can choose (Ramsey solution).

Operators. Network operators are profit maximizers. We assume that operators invest in different regions, i.e., network infrastructures are non-overlapping.[16] The network installation cost (i.e., the cost of covering cities by fibre) is convex in the sense that it is more expensive to connect subscribers in peripheral cities.[17] This captures the fact that operators start investing from cities where fibre is relatively cheaper to install. For sake of simplicity, we assume that the investment cost follows the form

$$C(I_i) = cI_i^2/2, \tag{2}$$

where $c > 0$ is a constant.[18]

We assume that subscribers pay independently of the traffic volume exchanged in the communications, i.e., they only pay for accessing the network, e.g. a periodical subscription fee. This reflects the fact that broadband offers are essentially flat rates. Let then p_i denote i's retail price to provide broadband access to one subscriber. The respective mass of subscribers using i's service in one city is denoted by q_i.

Network i faces a marginal cost, per subscriber, for serving broadband access equal to

$$\begin{cases} 0, & \text{if subscriber in } i\text{'s area} \\ a_j, & \text{if subscriber in } j\text{'s area.} \end{cases}$$

Subscribers. For each city covered by fibre we assume a "Hotelling model with hinterlands" specification regarding subscribers' choices.[19]

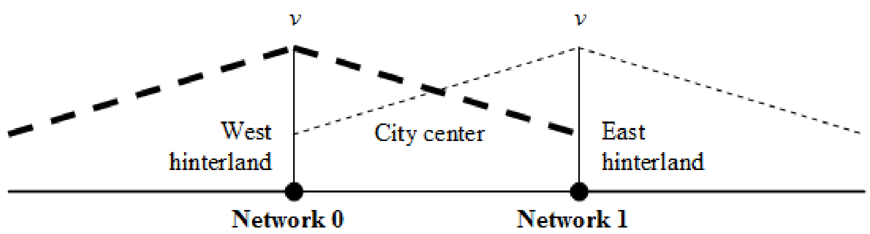

Figure 1: Hotelling model with hinterlands - the representative city. Consumers' willingness to pay for broadband service offered by network 0 and 1 represented by bold and thin dashes, respectively.

From the subscriber perspective there is some service differentiation among networks

[16]For example, if $I_0 = 10$ and $I_1 = 2$, we interpret this as operator 0 covering ten cities in the north part of the country; while operator 1 covers two cities in the south part of the country. It is implicitly assumed that cities are identical with regard to their population, however they have a different cost of fibre coverage.

[17]See Foros and Kind (2003). The convexity of costs also applies to postal services and third generation mobile telephone systems.

[18]Results in the paper are not dependent on the quadratic form of $C(I_i)$. Results will hold as long as networks' profits are concave in investment, i.e., $C(I_i)$ is sufficiently convex.

[19]See the section *Mobile market expansion* in Amstrong and Wright (2009) for another application of the Hotelling model with hinterlands.

for reasons such as technical support, proximity to clients, marketing campaigns, advertising or changing costs. Each city is composed by the center plus two symmetric hinterlands (West and East sides of the city center) as in Figure 1. Subscribers located in the city center, indexed by $\tilde{x} \in [0,1]$, are all served, while consumers in the hinterlands, indexed by $\tilde{y}$, are served as long as they are close enough to the center. In a representative city the surplus of a consumer indexed by $\tilde{x}$ and $\tilde{y}$ is defined by, respectively, $CS_{\tilde{x}}$ and $CS_{\tilde{y}}$

$$CS_{\tilde{x}} \equiv \begin{cases} v - t\tilde{x} - p_0 & \text{if operator } 0 \\ v - t(1-\tilde{x}) - p_1 & \text{if operator } 1 \end{cases} \tag{3}$$

$$CS_{\tilde{y}} \equiv \begin{cases} v - \tau\tilde{y} - p_i & \text{if operator } i = 0,1 \\ 0 & \text{if no service} \end{cases} \tag{4}$$

where v is the intrinsic value from subscribing to the service and t and τ represent the subscribers' disutility in the city center and hinterlands, respectively, from not being connected to their ideal network. We assume that $v > t = \tau$, i.e., service differentiation must be sufficiently small as compared to the intrinsic value v. There is a total mass two of consumers in a representative city. In the city center represented by the unit interval $[0,1]$ there is a mass 1 of consumers uniformly distributed with density 1, while in each hinterland there is a mass 1/2 of consumers uniformly distributed with density $\tau/2v$ on intervals $\left[0, \frac{v}{\tau}\right]$.[20] Gross consumer surplus (utility), U, and consumer surplus, CS, in a city is thus

$$U(x_0, z_0, x_1, z_1) = v(x_0 + x_1 + z_0 + z_1) - \frac{t(x_0^2 + x_1^2) + 2v(z_0^2 + z_1^2)}{2} \tag{5}$$

$$CS \equiv U - \sum_{i=0}^{1} p_i (x_i + z_i) \tag{6}$$

where x_i and z_i denote the mass of subscribers located in the city center and hinterlands, respectively, using network i's service. Note that $z_i \equiv y_i \frac{\tau}{2v}$, where y_i is the distance to the city center and z_i may be interpreted as the mass of subscribers, using network i's service, along that distance. Since consumers are fully served in the city center we have then $x_0 + x_1 = 1$.

The individual consumer surpluses from (3) and (4) imply that

$$x_i = \frac{1}{2} - \frac{p_i - p_j}{2t},\ z_i = \frac{v - p_i}{2v} = \frac{1}{2} - \frac{p_i}{2v}. \tag{7}$$

Therefore,

$$q_i \equiv x_i + z_i = 1 - \frac{(v+t)p_i - vp_j}{2tv} \text{ and } Q \equiv q_0 + q_1 = 2 - \frac{p_0 + p_1}{2v}. \tag{8}$$

[20] By assuming density $\tau/2v$ in the hinterlands we guarantee a fixed mass 1/2 of consumers in each hinterland. Otherwise the number of consumers in hinterlands would depend on τ and v.

A summary of the model's notation follows in Table 2.

Table 2: Notation

a_i	Access price, per subscriber, followed by network i.
x, y	Regulatory parameters with the indexation rule.
v	Intrinsic value from subscribing a fibre service.
t	Service differentiation parameter.
c	Investment cost parameter.
I_i	Number of cities covered by fibre installed by network i.
I	Total number of cities covered by fibre, defined as $I \equiv I_0 + I_1$.
p_i	Retail price charged by network i for broadband service.
x_i	Number of subscribers located in the city center and using network i's service.
z_i	Number of subscribers located in the hinterlands of a city using network i's service.
q_i	Total number of subscribers using network i's service in a city, defined as $q_i \equiv x_i + z_i$.
Q	Total number of broadband subscribers in a city, defined as $Q \equiv q_0 + q_1$.
U	Gross consumer surplus in a city.
CS	Consumer surplus in a city.

2.1 The social optimum benchmark

In order to assess the fixed and the indexation access price rules from the social standpoint we compute, as benchmark, the first-best solution that a benevolent planner could achieve. The social value of providing fibre access is equal to the sum of consumer surpluses, CS, in all cities covered by fibre, plus operators' subscription revenues minus the costs with regard to fibre coverage. Access prices are mere transfers among networks, therefore access revenues minus the access costs across operators sum up to zero. For that reason access prices are not relevant in the first-best analysis. In other words, the measure of social welfare taken is the unweighted sum of consumer surplus in all cities covered by fibre and the industry profit.

Given that the cities covered by fibre are identical, x_0, z_0, x_1, z_1 must be the same across them. Hence, in the first-best a benevolent regulator would solve

$$\begin{aligned} \max_{x_0,z_0,x_1,z_1,I_0,I_1} W &\equiv (I_0 + I_1)\,U - c\left(\frac{I_0^2}{2} + \frac{I_1^2}{2}\right) \\ \text{subject to } x_0 + x_1 &= 1. \end{aligned} \tag{9}$$

From the first-order conditions (FOCs) of the problem follows that

$$\begin{cases} x_i^{opt} = \frac{1}{2},\ z_i^{opt} = \frac{1}{2} \\ p_i^{opt} = 0 \\ I_i^{opt} = \frac{1}{4c}(6v - t) \\ U^{opt} = \frac{1}{4}(6v - t) \\ W^{opt} = \frac{(6v-t)^2}{16c} \end{cases}. \tag{10}$$

The efficient retail prices correspond to the social marginal cost of serving a fibre subscriber, i.e., zero by assumption. Thus, it is socially optimal to supply FTTH to all consumers in a representative city. Due to symmetry of willingness to pay for service between networks, the welfare-maximizing market shares in the city center and hinterlands are given by $x_i^{opt} = 1/2$ and $z_i^{opt} = 1/2$, respectively. With regard to investment, from a social standpoint, subscriber valuations v and t are driving the efficient network size, as well as the investment cost parameter c. It is noteworthy that in the absence of lump-sum transfers the social optimum is not feasible under any access price rule per subscriber. In the social optimum $p_i^{opt} = 0$, therefore networks would not extract revenues from subscribers, while the access revenue $aq_j^{opt} I_i^{opt}$ is equal to access cost $aq_i^{opt} I_j^{opt}$ under symmetry. Profits would be then negative

$$\Pi_i^{opt} = I^{opt} \times p_i^{opt} q_i^{opt} + aq_j^{opt} I_i^{opt} - aq_i^{opt} I_j^{opt} - c\left(I_i^{opt}\right)^2/2 = -c\left(I_i^{opt}\right)^2/2 < 0$$

and networks would prefer to exit the market. Therefore, the first-best solution is not feasible without lump-sum transfers that could cover the networks' investment cost. We can conclude that maximizing social welfare subject to non-negative profits, the Ramsey price must be then strictly positive.

Bearing in mind the first-best benchmark in (10) we establish, in the following section, the comparison to the market solutions under: a fixed access price, an indexation access price rule and the regulatory holidays regime.

3 The Subgame Perfect Nash Equilibria

In this section we explore the market outcomes when networks operate under a fixed access price, an indexation access price rule and the regulatory holidays regime. The model is solved by backward induction to find the symmetric subgame perfect Nash equilibrium. First, we solve the networks' problem for profit maximizing retail prices, given the investment levels. Second, we solve the networks' problem for optimal investments. Third, we solve the regulator's problem for welfare-maximizing access pricing rules (fixed and indexed access prices). Technical details and calculations follow in the appendix.

3.1 The fixed access price approach

We start by presenting and solving the networks' problems under the fixed access price rule. Then, we claim the inefficiencies of the fixed access price rule and run a numerical example as illustration.

3.1.1 Stage III: retail price choices

In the retail pricing stage operator i's problem is, given the access price a, (I_i, I_j) and p_j,

$$\max_{p_i} \Pi_i = I \times p_i q_i + a q_j I_i - a q_i I_j - c I_i^2/2, \tag{11}$$

where $I \times p_i q_i$ represents the subscription revenues and $a q_j I_i - a q_i I_j$ denotes the access revenue charged to network j minus the access payment for serving i's subscribers located in cities covered by j. The term $c I_i^2/2$ corresponds to the investment costs of covering I_i cities by fibre. From the FOC, in equilibrium we get

$$p_i^* = \frac{\left(3v\left(2t+a\right)+4t^2\right)v\left(I_i+I_j\right)+at\left(v\left(3I_i+4I_j\right)+2tI_j\right)}{\left(2t+v\right)\left(2t+3v\right)\left(I_i+I_j\right)}, \; i,j=0,1,\; j\neq i, \tag{12}$$

as long $p_i^*\left(a\right) \leq v - t/2$. In plain words, the equilibrium price must be below the willingness to pay of the middle consumer in the city center. Otherwise the full coverage assumption of the center would not hold. Assuming investment symmetry, the price equilibrium in (12) is then valid under constraint $a \leq \bar{a} \equiv \left(2v^2 - tv - 2t^2\right)/\left(t+2v\right)$. Plugging (12) into (8) we get

$$q_i^* = \frac{\left(6v^2+10tv+4t^2\right)v\left(I_i+I_j\right)-2a\left(t^2I_j+v^2I_i\right)-4v^2aI_j-atv\left(I_i+6I_j\right)}{2v\left(2t+v\right)\left(2t+3v\right)\left(I_0+I_1\right)} \tag{13}$$

and

$$Q^* = \frac{4v\left(t+v\right)-a\left(t+2v\right)}{2v\left(2t+v\right)}.$$

3.1.2 Stage II: investment choices

In the investment stage, with a fixed access price a, network i's maximization problem is, given I_j,

$$\max_{I_i} \Pi_i^* = I \times p_i^* q_i^* + a q_j^* I_i - a q_i^* I_j - c I_i^2/2,$$

where p_i^* is defined by (12) and q_i^* by (13) in the previous stage.

From the FOC of networks' problem, $\partial\Pi_i^*/\partial I_i = 0$, in equilibrium we reach

$$I_i^* = \frac{(t+2v)\left(48tv^2+32t^2v+24v^3-a\left(25tv+14t^2+12v^2\right)\right)a+16\left(t+v\right)\left(2t+3v\right)tv^2}{8cv\left(2t+3v\right)\left(2t+v\right)^2}. \tag{14}$$

3.1.3 Stage I: access price regulation

Here we compare the first-best to the market solution with a fixed access price rule. We claim (Proposition 1) that under the fixed access price rule it is not possible a regulator to implement the socially optimal investment, regardless of how much static efficiency is sacrificed.

Proposition 1 (underinvestment) *Under a fixed access price (i) is not possible to implement the socially optimal investment, i.e., there is underinvestment $I_i^* < I_i^{opt}$, and (ii) retail price efficiency requires a negative access price.*
Proof: Technical details follow in appendix for all the proofs.

The intuition for the underinvestment result with a fixed access price comes from the fact that networks are unable to capture the full social benefit of investment. This inability stems from (i) retail price competition and (ii) uniform pricing. The fixed access price rule is a regulatory tool that can decrease retail price competition in order to increase investment rewards. Nonetheless, even when the access price is set to maximize the investment outcome by reducing retail price competition, there are benefits captured by subscribers due to their heterogeneity in the willingness to pay for the service and the fact that networks are unable to price discriminate to extract the subscribers' full valuations. Moreover, even if networks were able to practice first-degree price discrimination, retail price competition would imply positive surplus to subscribers. Hence, networks do not internalize the full benefits from investments implying a choice that is necessarily inefficient. With regard to retail price (in)efficiency, due to the existence of market power in the downstream market, the access price would have to be negative to counterbalance the market power effect. In a nutshell, considering a non-negative fixed access price, the market outcome is condemned to underinvestment and retail price inefficiency. This result holds regardless the access price is bargained between firms in an unregulated market or it is set by a benevolent regulator. We can show that Proposition 1 is valid under a set of more general assumptions.

Theorem 1 (underinvestment) *Consider a sequential game such that the regulator chooses the access price a^* before networks compete first in investments and second in retail prices, and the following conditions hold: (a) network i's profit is defined by $\Pi_i = (I_i + I_j) \times p_i q_i + a_i q_j I_i - a_j q_i I_j - C(I_i)$, where $C(I_i)$ is an increasing and sufficiently* *convex cost function assuring strict concavity of profit in I_i; (b) social welfare measure is $W \equiv \sum_{i=0}^{1} [\Pi_i + I_i \times CS]$ where $\frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j^*}{\partial I_i} - \frac{\partial \Pi_j}{\partial I_i} < CS \equiv U - \sum_{i=0}^{1} p_i q_i$, (c) $q_i(p_i, p_j)$ is twice differentiable and non-increasing in $(p_i, -p_j)$.*

Thus, under a fixed access price (i) is not possible to implement the socially optimal investment, i.e., there is underinvestment $I_i^* < I_i^{opt}$, and (ii) retail price efficiency requires a negative access price.

We consider below a simple numerical example to determine the best that the regulator can do with a fixed access price.

A numerical example (part I): Suppose that $v/100 = t = c = 1$ and the regulator's goal is to maximize social welfare. Under the fixed access price methodology a regulator solves $\max_a W$ subject to equations (12), (13) and (14).[21] Plugging the three previous restrictions into the social welfare function, the regulator's problem under the fixed access price rule is depicted in Figure 2.

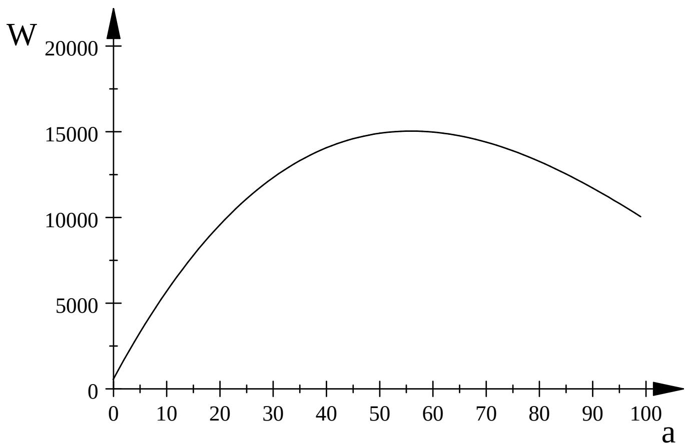

Figure 2: Welfare function for $v/100 = t = c = 1$, $a \leq \bar{a} = 99$.

In order to assure that the city center is fully served, i.e., $p_i^*(a) < v - t/2 = 99.5$, the access price must be below 99. The welfare-maximizing access price is $a^* = 55.9$ and in equilibrium

$$
\begin{aligned}
I_i^* &= 80.8 < 149.8 = I_i^{opt}, && (15)\\
p_i^* &= 57.1 > 0 = p_i^{opt}, && (16)\\
q_i^* &= 0.715 < 1 = q_i^{opt}, && (17)\\
U^* &= 133.5 < 149.8 = U^{opt}, && (18)\\
W^* &= 15039 < 22425 = W^{opt}. && (19)
\end{aligned}
$$

The inequality in (15) illustrates the underinvestment problem claimed in Proposition 1. The inequality in (16) is due to networks' market power that drives retail prices to

[21] The regulator's problem is subject to networks' break-even constraint. In this numerical example this restriction is not binding, $\Pi_i^* = 3325.8$, being discarded from the analysis.

a level above the marginal cost. Consequently, the mass of subscribers is lower than in the first-best solution, as illustrated by (17), and gross consumer surplus decreases as compared to the social optimum, pointed by (18). Since operators underinvest and retail prices are above marginal cost, generating consumption distortions, social welfare is thus strictly below the first-best level, as represented by inequality (19).

3.2 The new rule: access price indexation

Given the fixed access pricing inefficiencies identified in Proposition 1, we consider a new rule, by indexing access prices to networks' investment choices, with the purpose of increasing investment incentives. In particular, the new access price rule is defined by (1) where $(x, y) \in \mathbb{R}_+^2$ is the pair of regulatory parameters to be determined. Under access price indexation, investments affect networks' profits through the changes in access prices, besides the extra retail and access revenues of owning a larger network covering more cities.

We solve the three-stage game under the new rule with a welfare-maximizing regulator (second-best analysis) and compare the outcome to the welfare-maximizing solution obtained under a fixed access price. We also solve the stage game for a regulator whose goal is to implement the socially efficient (first-best) level of fibre coverage with the lowest possible retail pricing. This illustrates that an access price indexation can be useful to achieve other goals (than welfare maximization) more efficiently than a fixed access price. We show in Proposition 2 that access price indexation can increase the social welfare relatively to a fixed access pricing. In Proposition 3 we show that indexation can promote the socially efficient investment, answering the question in the title.

3.2.1 Stage III: retail price choices under indexation

With access price indexation, operator i's optimization problem in the retail pricing stage is, given (I_i, I_j) and p_j

$$\max_{p_i} \Pi_i = I \times p_i q_i + a_i q_j I_i - a_j q_i I_j - c I_i^2/2. \tag{20}$$

Note that the problem in (20) is different from the one in (11) since access prices may now differ among operators depending on investment levels.

Taking the FOC of the problem in (20) and solving for the equilibrium retail prices we get

$$p_i^{**} = \frac{v\left(3ta_i + 3va_i + 6tv + 4t^2\right) I_i + \left(6tv^2 + 4t^2 v + 2t^2 a_j + 3v^2 a_j + 4tva_j\right) I_j}{(2t + v)(2t + 3v)(I_i + I_j)}. \tag{21}$$

Plugging (21) into (7) and (8) we reach

$$\begin{cases} x_i^{**} = \frac{(2t+3v+a_i)I_i+(2t+3v-a_j)I_j}{2(I_i+I_j)(2t+3v)} \\ z_i^{**} = \frac{v\left(2tv+3v^2-3ta_i-3va_i\right)I_i+\left(2tv^2-2t^2a_j-3v^2a_j-4tva_j+3v^3\right)I_j}{2v(2t+v)(2t+3v)(I_i+I_j)} \\ q_i^{**} = \frac{v\left(10tv+4t^2+6v^2-ta_i-2va_i\right)I_i+2(t+v)\left(2tv+3v^2-ta_j-2va_j\right)I_j}{2v(2t+v)(2t+3v)(I_i+I_j)} \end{cases}. \tag{22}$$

3.2.2 Stage II: investment choices under indexation

In the investment stage, network i's maximization problem is, given a_i, a_j and I_j,

$$\max_{I_i} \Pi_i^{**} = I \times p_i^{**} q_i^{**} + a_i q_j^{**} I_i - a_j q_i^{**} I_j - c I_i^2/2,$$

where p_i^{**} is defined by (21) and q_i^{**}, and analogously q_j^{**}, by (22). The network's optimal investment is characterized now by

$$\tfrac{d\Pi_i^{**}}{dI_i} \equiv \underbrace{\frac{\partial \Pi_i^{**}}{\partial I_i}}_{\text{“direct effect”}} + \underbrace{\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i}}_{\text{“indexation effect”}} = 0,$$

while under the fixed access approach only the “direct effect” exists. The “direct effect” means that operators account for the extra retail and access revenues and direct costs of owning a larger network covering more cities. Therefore, this effect is present under both the access price methodologies. The “indexation effect” accounts for the profit variation through the changes in the access price when operators vary their investments. Under the fixed access price $a_k = a^*$ does not depend on investments, then $\partial a_k / \partial I_i = 0$ and the “indexation effect” vanishes. Under the new access price rule the same effect is positive (see the proof of Proposition 2). In particular, for $(x, y) \in \mathbb{R}_+^2$,

$$\underbrace{\frac{\partial \Pi_i^{**}}{\partial a_i}}_{(+)} \underbrace{\frac{\partial a_i}{\partial I_i}}_{(+)} + \underbrace{\frac{\partial \Pi_i^{**}}{\partial a_j}}_{(-)} \underbrace{\frac{\partial a_j}{\partial I_i}}_{(-)} > 0$$

meaning that network i's profit increases (decreases) if i charges (pays) higher access prices, and the access price charged (paid) increases (decreases) in i's investment. This implies that under the indexation approach the “direct effect” must be negative, whilst under the fixed access approach the same effect is zero. For any access price $a_i = a^* > 0$, $\partial \Pi_i^{**} / \partial I_i = \partial \Pi_i^{*} / \partial I_i$. Thus, by concavity of Π_i^* with respect to I_i, networks choose to invest more with the indexation approach than under the fixed access pricing.

Assuming investment symmetry, operator i's FOC for investment can be written as

$$\left\{\begin{array}{c} 3\left(t+2v\right)\left(y-x\right)\left[\left(10\left(t^2+v^2\right)+19tv\right)x-\left(2t+3v\right)\left(t+2v\right)y\right]I_i^2+ \\ -8v\left[\left(2t+3v\right)\left(c\left(2t+v\right)^2+v\left(t+2v\right)y\right)-2\left(t+2v\right)\left(6tv+4t^2+3v^2\right)x\right]I_i \\ +16tv^2\left(t+v\right)\left(2t+3v\right) \end{array}\right\}=0, \tag{23}$$

while the following inequality has to be satisfied for the second-order condition (SOC) to hold,

$$S\equiv-\left\{\begin{array}{c} 8v\left(2t+3v\right)\left[c\left(2t+3v\right)\left(2t+v\right)^2-2\left(t+2v\right)\left(6tv+4t^2+3v^2\right)x\right]+ \\ +\left(t+2v\right)\left(\begin{array}{c} \left(132t^3+162v^3+433tv^2+404t^2v\right)x^2+ \\ +\left(t+2v\right)\left(2t+3v\right)^2y^2-6\left(2t+3v\right)\left(19tv+10t^2+10v^2\right)xy \end{array}\right)I_i \end{array}\right\}<0. \tag{24}$$

3.2.3 Stage I: the second-best

The welfare-maximizing regulator solves the following problem

$$\max_{x,y} W\equiv\left(I_i+I_j\right)U-c\left(I_i^2+I_j^2\right)/2 \text{ subject to} \tag{25}$$
$$x_i^{**}\left(I_i,I_j,x,y\right)=x_i^{**} \text{ (Stage III)}$$
$$z_i^{**}\left(I_i,I_j,x,y\right)=z_i^{**} \text{ (Stage III)}$$
$$q_i^{**}\left(I_i,I_j,x,y\right)=q_i^{**} \text{ (Stage III)}$$
$$p_i^{**}\left(I_i,I_j,x,y\right)=p_i^{**} \text{ (Stage III)}$$
$$d\Pi_i^{**}/dI_i=0 \text{ (Stage II)}$$
$$d^2\Pi_i^{**}/dI_i^2\leq 0 \text{ (Stage II)}$$
$$\Pi_i^{**}\geq 0 \text{ (PC)},$$

where PC denotes network i's participation constraint. Plugging the restrictions from stage III into the objective function in (25) and assuming investment symmetry, the regulator's problem under the indexation approach can be rewritten as

$$\max_{x,y} W=I_i\frac{\left(\begin{array}{c} 2v\left(23tv^2+12t^2v-4t^3+6v^3\right)+ \\ -4v\left(c\left(2t+v\right)^2+2t\left(t+2v\right)\left(x-y\right)\right)I_i+ \\ -\left(x-y\right)^2\left(t+2v\right)^2I_i^2 \end{array}\right)}{4v\left(2t+v\right)^2} \text{ subject to}$$
$$I_i\frac{16tv^2\left(v+t\right)-2v\left(4ct\left(v+t\right)+cv^2+v\left(2t+4v\right)\left(y-x\right)\right)I_i-\left(x-y\right)^2\left(t+2v\right)^2I_i^2}{4v\left(2t+v\right)^2}\geq 0 \text{ (PC)}$$
$$d\Pi_i^{**}/dI_i=0 \text{ (Stage II)}$$
$$d^2\Pi_i^{**}/dI_i^2\leq 0 \text{ (Stage II)}.$$

Lemma 1 (participation constraint binds) *Networks' participation constraint will be binding, i.e.,* $\Pi_i^{SB} = 0$*, with a welfare-maximizing regulator using the access price indexation.*

Recall that in the social optimum networks' would have negative profits. Given that the social optimum is not feasible in the absence of transfers, the best that a welfare-maximizing regulator can do under the indexation approach is to choose a regulatory regime (x, y) such that networks' profits are nil. If networks presented positive profits, the regulator could enhance the social welfare by choosing (x, y) such that retail prices were lower and investments in fibre coverage higher.

Proposition 2 (indexation vs fixed access) *A linear access pricing rule depending on investments with* $(x, y) \in \mathbb{R}_+^2$ *can simultaneously (i) expand total investment in fibre coverage, (ii) expand the mass of subscribers in each city and (iii) enhance social welfare, as compared to a fixed access price* $a^* > 0$*.*

The introduction of the access price indexation creates a scheme of rewards to investors and punishment to those who do not invest or invest relatively less. In particular, networks that invest more will benefit from a lower access price when accessing other network, and may charge a higher price when providing access. As a result of the access price indexation, networks choose to invest more than under a fixed access price.

The mass of subscribers depends on the retail price level which in turn depends on the access price level. Therefore, if the access price with the indexation rule is below the one defined by the fixed access price rule, there will be more fibre subscribers under the former rather than under the latter. Suppose that with the fixed access price rule $a_i = a^*$. Under the access price indexation the regulator can choose (x, y) such that $a_i = xI_i - yI_j < a^*$. With investment symmetry in equilibrium, the last inequality is equivalent to $x - y < a^*/I_i$. In a nutshell, x and y can be defined at any level with the purpose of providing incentives for higher investment levels, as long as the difference $x - y$ is sufficiently small to assure that the access price is smaller as compared to the fixed access approach.

With regard to social welfare, the mass of subscribers in each city increases when implementing the indexation approach relatively to the fixed access approach. Then, the gross consumers surplus in each city must be higher under the former approach. Given that in the first-best $\partial W/\partial I_i = U - cI_i = 0$, in the second-best $U - cI_i \geq 0$, otherwise the regulator could increase welfare by providing less incentives to invest. Hence, the social welfare variation by implementing the access price indexation must be positive. This is explained with the increase of gross consumers surplus in each city together with the increase in the number of cities covered by fibre. We show that Proposition 2 is robust to a set of more general assumptions.

Theorem 2 (indexation vs fixed access) *Consider a sequential game such that the* *regulator chooses the access price a_i before networks compete first in investments and second in retail prices, and the following conditions hold: (a) network i's profit is defined by $\Pi_i = (I_i + I_j) \times p_i q_i + a_i q_j I_i - a_j q_i I_j - C(I_i)$, where $C(I_i)$ is an increasing and sufficiently convex cost function assuring strict concavity of profit in I_i; (b) $\partial \Pi_i / \partial p_j \geq 0$* in equilibrium $(p_i, p_j) = \left(p_i^{**}, p_j^{**}\right)$*; (c) Π_i is strictly concave in p_i; (d) $q_i(p_i, p_j)$ is twice differentiable and non-increasing in $(p_i, -p_j)$; (e) social welfare measure is $W \equiv \sum_{i=0}^{1} [I_i U - C(I_i)]$; (f) gross consumer surplus, U, in each city is decreasing in $(p_i, p_j) \in \mathbb{R}_+^2$.*

Thus, an access pricing rule depending on investments can simultaneously (i) expand total investment in fibre coverage, (ii) expand the mass of subscribers in each city and (iii) enhance social welfare, as compared to a fixed access price $a^* > 0$.

Below we furnish Proposition 2 with a numerical example.

A numerical example (part II): Solving the regulator's problem in (25) for $v/100 = t = c = 1$ the regulator chooses $(x, y) = (0.352, 0.0537)$ and announces the following access price rule[22]

$$a_i = 0.352 I_i - 0.0537 I_j, \tag{26}$$

that induces to the following (second-best (SB)) equilibrium,

$$\begin{aligned}
I_i^{SB} &= 124 > 80.8 = I_i^*, \\
a_i^{SB} &= 36.9 < 55.9 = a^*,\ p_i^{SB} = 38.4 < 57.1 = p_i^*, \\
q_i^{SB} &= 0.808 > 0.715 = q_i^*,\ U^{SB} = 142.4 > 133.5 = U^*, \\
\Pi_i^{SB} &= 0 < 3325.8 = \Pi_i^*,\ W^{SB} = 19938 > 15039 = W^*.
\end{aligned}$$

This numerical example illustrates that a linear access price on investments may incentive to more investment, lower retail prices and higher social welfare than a fixed access price. The total mass of subscribers increases by more than 73%, and social welfare increases by more than 32% as compared to the second-best under a fixed access price.

We show below that with the indexation rule, contrarily to the fixed access price, it is possible to achieve the socially efficient (first-best) investment level.

3.2.4 Stage I revisited: implementing the first-best investment level

Suppose that the regulator chooses a regulatory policy (x, y) with the purpose of implementing the first-best investment level $I_i = I_i^{opt} = (6v - t) / (4c)$. We claim that if

[22] Table A.1 in the appendix contains the numerical simulations. This table suggests that if the regulator is more concerned about investment (as opposed to retail price) efficiency, parameters x and y will be set at higher levels.

broadband service differentiation among operators is sufficiently small, then there will exist a regulatory policy (x, y) such that the first-best investment level I_i^{opt} can be implemented.

Proposition 3 (first-best investment level) *If service differentiation, t, is sufficiently small, a linear access pricing rule depending on investments can induce the market outcome to implement the first-best investment level.*

A regulatory regime (x, y) will implement the first-best investment, $I_i = I_i^{opt}$, if it passes three tests: (i) the network FOC in (23) (ii) the SOC, whose signal is defined by (24), and (iii) non-negative profits. In equilibrium, for $I_i = I_i^{opt}$, the regulator chooses (x, y) such that networks have zero profits and simultaneously satisfy the FOC. A small differentiation parameter is a sufficient (but not necessary) condition to ensure that the SOC is satisfied in networks' problems. Intuitively, if service differentiation is small it implies fiercer price competition between the two networks. Since price competition is more intense, a price cut is more valuable to networks because it shifts an increasing mass of subscribers towards the one that has cut the price. Therefore, if competition is fiercer, operator i will have more incentives to invest under the indexation approach as a means to inflate a_i and reduce a_j, achieving a competitive advantage at the retail pricing stage. As service differentiation decreases, it will be easier for the regulator to ensure the implementation of higher investment levels, namely the first-best investment level. We show that Proposition 3 holds under more general assumptions.

Theorem 3 (first-best investment implementation) *Consider a sequential game such that the regulator chooses the access price a_i before networks compete first in investments and second in retail prices, and the following conditions hold: (a) network i's profit is defined by $\Pi_i = (I_i + I_j) \times p_i q_i + a_i q_j I_i - a_j q_i I_j - C(I_i)$, where $C(I_i)$ is an increasing and sufficiently convex cost function to assure strict concavity of profit in I_i; (b) social welfare measure is $W \equiv \sum_{i=0}^{1} [\Pi_i + I_i \times CS]$ where $0 < CS \equiv U - \sum_{i=0}^{1} p_i q_i$, (c) $q_i (p_i, p_j)$ is twice differentiable and non-increasing in $(p_i, -p_j)$.*

Thus, as long as profits are non-negative, an access pricing rule depending on investments can induce the market outcome to implement the first-best investment level.

Follows a numerical example of the mechanics behind the result.

A numerical example (part III): Suppose that $v/100 = t = c = 1$ and the regulator's objective is to implement the first-best investment $I_i^{**} = I_i^{opt} = 149.8$ with the lowest possible retail price, p_i^{**}. For $I_i^{**} = I_i^{opt} = 149.8$ networks break-even (participation constraint binds) at $p_i^{**} = 49.9 < p_i^* = 57.1$. In order to implement the first-best investment the regulatory regime (x, y) must satisfy the FOC in (23) and the SOC, whose signal is defined by (24), for $I_i^{**} = I_i^{opt}$. Additionally, in order to induce the retail price

$p_i^{**} = 49.9$, by (21) the access charge in equilibrium must be equal to $a_i^{**} = 48.6$. Hence, the regulatory regime (x, y) must satisfy

$$a_0^{**} = a_1^{**} = 48.6 \Leftrightarrow 48.6 = (x - y) \times 149.8 \Leftrightarrow y = x - 0.325.$$

Graphically, the solution to the regulator's problem is given by the intersection of the two thick black lines in Figure 3.

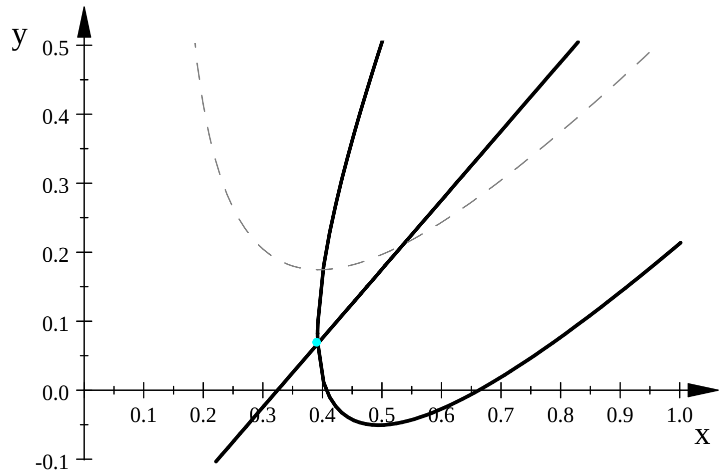

Figure 3: Operators' FOC (thick black curve) and zero-SOC (dashed gray curve) both evaluated at $I_i = 149.8$. The networks' SOCs are satisfied in the area below of dashed gray curve. The straight black line assures that $p_i^{**} = 49.9$.

The thick black curve corresponds to the set of regulatory regimes (x, y) such that the networks' FOCs are satisfied for $I_i^{**} = I_i^{opt}$, and the straight line is the set of regulatory regimes (x, y) such that the equilibrium retail price is $p_i^{**} = 49.9$ when $I_i^{**} = I_i^{opt}$. The SOC for the networks' problem is satisfied for all the regulatory regimes (x, y) below the dashed gray curve. Thus, at the intersection point of the thick black lines $(x, y) = (0.392, 0.0676)$ the SOC is fulfilled. The regulator announces then the access price rule

$$a_i = 0.392 I_i - 0.0676 I_j \tag{27}$$

and the market outcome is

$$\begin{aligned} I_i^{**} &= I_i^{opt} = 149.8 > 80.8 = I_i^*,\ p_i^{**} = 49.9 < 57.1 = p_i^*, \\ q_i^{**} &= 0.75 > 0.715 = q_i^*,\ W^{**} = 18700 > 15039 = W^*. \end{aligned}$$

In this numerical exercise we have shown that the implementation of the regulatory regime $(x, y) = (0.392, 0.0676)$ under the access price indexation rule expands the total number of cities covered by fibre until the first-best level. Moreover, such expansion is achieved without the need to increase retail prices relatively to the fixed access price methodology. The indexation rule can create further incentives to networks invest as compared to a fixed access price. The extra incentive consists in the possibility of networks gain a competitive advantage in the downstream market when they invest more. Thus, the appropriate calibration of the indexation rule, as opposed to the fixed access price approach, can induce the market outcome to achieve the first-best investment.

3.3 Regulatory holidays

In this section we show that the indexation rule can perform better than the regulatory holidays with regard to fibre coverage, retail prices and social welfare.[23] In a city monopolized by operator i, which is unable to price discriminate, the demand function faced by the monopolist serving both hinterlands is defined by[24]

$$q_i = \begin{cases} 2\left(\frac{v-p_i}{2v} + \frac{v-p_i}{t}\right) & \text{if } v \geq p_i > v - \frac{t}{2} \\ \frac{v-p_i}{v} + 1 & \text{if } p_i \leq v - \frac{t}{2} \end{cases}.$$

Operator i chooses p_i and I_i solving the following maximization problem

$$\max_{p_i, I_i} \Pi_i^{mon} = I_i p_i q_i - c I_i^2/2.$$

We demonstrate in the appendix that each monopoly network chooses to charge the retail price $p_i^{mon} = v - t/2$ serving $q_i^{mon} = 1 + t/(2v)$ subscribers in each city. Each network covers $I_i^{mon} = (4v^2 - t^2)/(4cv)$ cities by fibre and attains a profit level of $\Pi_i^{mon} = (2v-t)^2 (2v+t)^2 / (32cv^2)$. The dual inefficiency of the monopoly with respect to retail prices and investments is clear since $p_i^{mon} = v - t/2 > 0 = p_i^{opt}$ and $I_i^{mon} = (4v^2 - t^2)/(4cv) < (6v - t)/(4c) = I_i^{opt}$, provided that $v > t$. In a nutshell, the retail price inefficiency derives from networks' market power, while investment inefficiency is due to part of the surplus generated by the fibre service being captured by consumers (given uniform pricing and no lump-sum transfers). We show that the indexation approach can do better than the regulatory holidays with respect to social welfare. We provide a numerical example of the mechanics behind Proposition 4.

Proposition 4 (regulatory holidays) *A linear access pricing rule depending on investments can simultaneously decrease retail prices and increase investment and social welfare levels as compared to the regulatory holidays regime (i.e., local monopolies).*

A numerical example (part IV): Following the numerical exercise for $v/100 = t =$

[23] The regulatory holidays case can also be seen as a special case of the indexation rule, for example, by setting $x = \infty$, $y = 0$. In this case, for any investment $I_i > 0$ the access price, under the indexation rule, becomes infinite which is equivalent to grant local monopolies.

[24] In the appendix we solve for both the cases where the monopolist offers one or two brands, i.e., is present in one or both hinterlands of a city. Assuming that $v > 2t$, the results in Proposition 4 hold regardless of the monopolist presence in one or both hinterlands. Here we present the two-brand case which only requires $v > t$ to verify Proposition 4.

$c = 1$, the monopoly outcome is

$$\begin{aligned}
I_i^{mon} &= 100 < 124 = I_i^{SB},\ p_i^{mon} = 99.5 > 38.4 = p_i^{SB}, \\
Q^{mon} &= 1.005 < 1.62 = Q^{SB},\ U^{mon} = 100.3 < 142.4 = U^{SB}, \\
\Pi_i^{mon} &= 4999.8 > 0 = \Pi_i^{SB},\ W^{mon} = 10050 < 19938 = W^{SB}.
\end{aligned}$$

Granting a local monopoly expands total investment relatively to a fixed access price but at the cost of a retail price distortion reducing the mass of subscribers. The regulatory holidays regime is dominated by the proposed access price indexation rule, both in terms of investment (broadband coverage by fibre) and retail prices, resulting in higher welfare with the indexation approach than with regulatory holidays. Intuitively, the regulatory holidays policy consists in alleviating competition pressure to increase the investment rewards as a way to encourage more investment. The indexation approach goes in the opposite direction proposing a "tournament" where networks have incentives to compete in investments.

4 Conclusions

Investment incentives have been at the core of the access debate. Some authors argue that networks will not invest in facilities subject to strong access regulation (e.g. Sidak and Spulber (1996) on open access). Others have supported the idea of forced access because of the gains in static efficiency, but advise that the access price must take into account investment incentives (e.g. Laffont and Tirole (2001)). This paper contributes to this debate with the formulation of a new rule for access pricing. We have shown that access pricing rules depending on the investment level of each network, without being informationally more demanding, can boost investment efficiency without sacrifice of retail price efficiency and ultimately enhance social welfare vis-à-vis the rules of fixed access price.

Under the proposed indexation rule operators are aware that by investing less they will pay (receive) a higher (lower) access price when competing in the downstream market. Free riders on network investment will be less competitive in the downstream market, thus being punished with a lower profit level relatively to whom invests more and consequently is awarded with a competitive advantage. By setting the appropriate indexation rule, regulators can open an important avenue for harsher competition in investment. We have shown that the proper calibration of the indexation rule can induce the market equilibrium to achieve the socially efficient investment, impossible to reach with a fixed access price. Moreover, the access price indexation can perform better in social welfare than granting access holidays. While granting access holidays consists in a temporary reduction of retail competition to stimulate investments, the indexation rule goes in the opposite way enticing competition among operators beginning from the investment stage.

Despite our model is placed within the NGNs context, namely the fibre deployment problem, the logic of our results goes beyond particular cases. In general, results herein presented are valid to any infrastructure facilities facing an underinvestment problem and whose operators have to choose non-cooperatively the investment levels and compete in retail prices or, equivalently, in quantities.

There are some issues which we do not address in this paper but that may be of potential interest for future research. First, we have assumed full information over the analysis, in particular in the decision-making process of the regulator. A question for future research is whether results will hold when the regulator faces informational constraints, e.g. uncertainty on a set of parameters with regard to demand or costs. We note, however, that the need for information it is just as much a problem for the indexation approach as it is for the fixed access approach. The estimation of the relevant parameters is inevitably imperfect, and estimation errors imply efficiency losses under both methodologies. Second, we do not model the entry decisions made by networks, as we assume, for sake of technical simplicity, that there are two symmetric networks. We believe though that results and intuitions on the indexation rule should extend to non-symmetric cases and to the N-operator case as well. Third, we do not consider what happens if the networks' facilities are subject to congestion. While this is not currently a concern for NGNs since these are considered high-speed networks, one may want to relax the non-rivalry assumption in applications to other infrastructures. Despite the shortcomings, this paper demonstrates the potential benefits of a new access pricing rule that welfare dominates both the regulatory holidays solution and the fixed access pricing methodology.

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6 Appendix

6.1 Hotelling Model with hinterlands

Short list of the main assumptions:
(i) In a given city, the surplus of a consumer indexed by $\tilde{x}$ and $\tilde{y}$ is defined by $CS_{\tilde{x}}$ and $CS_{\tilde{y}}$, respectively, where

$$
\begin{aligned}
CS_{\tilde{x}} &\equiv \begin{cases} v - t\tilde{x} - p_0 & \text{if operator } 0 \\ v - t(1-\tilde{x}) - p_1 & \text{if operator } 1 \end{cases} \\
CS_{\tilde{y}} &\equiv \begin{cases} v - \tau\tilde{y} - p_i & \text{if operator } i = 0, 1 \\ 0 & \text{if no service} \end{cases}.
\end{aligned}
$$

(ii) Each city is composed by the center and two hinterlands (West and East side of the city center). In the city center there is a mass 1 of consumers (indexed by $\tilde{x}$) uniformly distributed with density 1 in the unit interval $[0, 1]$. Each hinterland has a mass $1/2$ of consumers (indexed by $\tilde{y}$) uniformly distributed with density $\tau/2v$.

The gross consumer surplus U and the consumer surplus CS in a representative city are, respectively

$$\begin{aligned}
U\left(x_0, y_0, x_1, y_1\right) &= \int_0^{x_0}(v-t\tilde{x})\,d\tilde{x}+\int_0^{y_0}(v-\tau\tilde{y})\frac{\tau}{2v}d\tilde{y}+\int_0^{x_1}(v-t\tilde{x})\,d\tilde{x}+\int_0^{y_1}(v-\tau\tilde{y})\frac{\tau}{2v}d\tilde{y} \\
&= v\left(x_0+x_1+z_0+z_1\right)-\frac{t\left(x_0^2+x_1^2\right)+2v\left(z_0^2+z_1^2\right)}{2}, \text{ and} \\
CS &\equiv U-\sum_{i=0}^{1} p_i\left(x_i+z_i\right),
\end{aligned}$$

where $z_i \equiv y_i \frac{\tau}{2v}$.
(iii) The city center is fully served, i.e., $x_0+x_1=1$.
(iv) I_i corresponds to the number of cities covered by fibre by operator i. The total number of cities covered by fibre is I, where $I \equiv I_0+I_1$.
(v) Investment cost for operator $i=0,1$, is given by technology

$$C(I_i)=cI_i^2/2.$$

(vi) The marginal cost of serving subscribers is zero (except access charges, when applicable).

6.1.1 The social optimum

$$\begin{aligned}
\max_{x_0,z_0,x_1,z_1,I_0,I_1} W &\equiv \left(I_0+I_1\right)U-c\left(I_0^2+I_1^2\right)/2 \\
\text{subject to } x_0+x_1 &= 1
\end{aligned}$$

FOC:

$$\left\{\begin{array}{c}
\frac{dW}{dx_0}=-t\left(I_0+I_1\right)\left(2x_0-1\right)=0 \\
\frac{dW}{dz_0}=-\left(I_0+I_1\right)v\left(2z_0-1\right)=0 \\
\frac{dW}{dz_1}=-\left(I_0+I_1\right)v\left(2z_1-1\right)=0 \\
\frac{dW}{dI_0}=-\frac{1}{2}\left(t-2v+2cI_0-2tx_0-2vz_0-2vz_1+2tx_0^2+2vz_0^2+2vz_1^2\right)=0 \\
\frac{dW}{dI_1}=-\frac{1}{2}\left(t-2v+2cI_1-2tx_0-2vz_0-2vz_1+2tx_0^2+2vz_0^2+2vz_1^2\right)=0
\end{array}\right.$$

$$\Leftrightarrow\left\{\begin{array}{c}
x_i^{opt}=\frac{1}{2}, \\
z_i^{opt}=\frac{1}{2}, \\
I_i^{opt}=\frac{1}{4c}\left(6v-t\right), \\
U^{opt}=\frac{1}{4}\left(6v-t\right), \\
W^{opt}=\frac{(6v-t)^2}{16c}.
\end{array}\right.$$

6.1.2 Consumer demand functions

The individual consumer surplus defined by (3) implies that,

$$x_i = \frac{1}{2} - \frac{p_i - p_j}{2t},$$

and from (4) we get

$$z_i = \frac{v - p_i}{\tau} \times \frac{\tau}{2v} = \frac{v - p_i}{2v}.$$

Hence,

$$q_i \equiv x_i + z_i = \frac{1}{2} - \frac{p_i - p_j}{2t} + \frac{v - p_i}{2v} = 1 - \frac{(v+t)\,p_i - vp_j}{2tv} \text{ and } Q = q_0 + q_1 = 2 - \frac{p_0 + p_1}{2v}.$$

6.1.3 The fixed access price approach

STAGE III: RETAIL PRICE COMPETITION

Operator 0's problem,

$$\begin{aligned} \max_{p_0} \Pi_0 &= (I_0 + I_1) \times p_0 q_0 + a q_1 I_0 - a q_0 I_1 - c I_0^2/2 \\ FOC &: \frac{atI_1 + v\,(a + 2t)\,(I_0 + I_1) - 2\,(t+v)\,(I_0 + I_1)\,p_0 + v\,(I_0 + I_1)\,p_1}{2tv} = 0. \end{aligned}$$

Operator 1's problem,

$$\begin{aligned} \max_{p_1} \Pi_1 &= (I_0 + I_1) \times p_1 q_1 + a q_0 I_1 - a q_1 I_0 - c I_1^2/2 \\ FOC &: \frac{atI_0 + v\,(a + 2t)\,(I_0 + I_1) - 2\,(t+v)\,(I_0 + I_1)\,p_1 + v\,(I_0 + I_1)\,p_0}{2tv} = 0. \end{aligned}$$

In equilibrium,

$$p_i^* = \frac{2at^2 I_j + 3av^2\,(I_i + I_j) + 6tv^2\,(I_i + I_j) + 4t^2 v\,(I_i + I_j) + 3atvI_i + 4atvI_j}{(2t+v)\,(2t+3v)\,(I_i + I_j)},$$

thus

$$\begin{aligned} q_i^* &= \frac{(6v^2 + 10tv + 4t^2)\,v\,(I_i + I_j) - 2a\,(t^2 I_j + v^2 I_i) - 4v^2 a I_j - atv\,(I_i + 6I_j)}{2v\,(2t+v)\,(2t+3v)\,(I_i + I_j)} \\ Q^* &= \frac{4tv + 4v^2 - at - 2av}{2v\,(2t+v)}. \end{aligned}$$

STAGE II: INVESTMENT

$$
\begin{aligned}
\max_{I_i} \Pi_i^* &= I \times p_i^* q_i^* + a q_j^* I_i - a q_i^* I_j - c I_i^2/2 \\
FOC &: \frac{d\Pi_i^*}{dI_i} = \frac{\partial \Pi_i^*}{\partial I_i} + \frac{\partial \Pi_i^*}{\partial a}\frac{\partial a}{\partial I_i} = 0
\end{aligned}
$$

In equilibrium,

$$
I_i^* = \frac{(t+2v)\left(48tv^2 + 32t^2v + 24v^3 - a\left(25tv + 14t^2 + 12v^2\right)\right)a + 16\left(t+v\right)\left(2t+3v\right)tv^2}{8cv\left(2t+3v\right)\left(2t+v\right)^2}.
$$

THE SOCIALLY EFFICIENT SOLUTION VS MARKET OUTCOME UNDER A FIXED ACCESS PRICE

The socially efficient solution is characterized by

$$
\begin{aligned}
I_i^{opt} &= \left(6v - t\right)/\left(4c\right), \\
x_i^{opt} &= 1/2,\ z_i^{opt} = 1/2,\ q_i^{opt} = 1,\ Q^{opt} = 2, \\
p_i^{opt} &= 0,\ U^{opt} = \left(6v - t\right)/4,\ W^{opt} = \left(6v - t\right)^2/\left(16c\right).
\end{aligned}
$$

Hence, in the socially efficient solution operators would present negative profits,

$$
\Pi_i^{opt} = I^{opt} \times p_i^{opt} q_i^{opt} + a q_j^{opt} I_i^{opt} - a q_i^{opt} I_j^{opt} - c\left(I_i^{opt}\right)^2/2 = -c\left(I_i^{opt}\right)^2/2 < 0.
$$

The market equilibrium under the fixed access price approach is characterized by

$$
\begin{aligned}
I_i^* &= \frac{(t+2v)\left(48tv^2 + 32t^2v + 24v^3 - a\left(25tv + 14t^2 + 12v^2\right)\right)a + 16\left(t+v\right)\left(2t+3v\right)tv^2}{8cv\left(2t+3v\right)\left(2t+v\right)^2}, \\
p_i^* &= \frac{1}{4t+2v}\left(at + 2av + 4tv\right),\ x_i^* = \frac{1}{2},\ z_i^* = \frac{2v^2 - \left(2v+t\right)a}{4v\left(2t+v\right)}, \\
q_i^* &= \frac{1}{2} + \frac{2v^2 - \left(2v+t\right)a}{4v\left(2t+v\right)},\ Q^* = 1 + \frac{2v^2 - \left(2v+t\right)a}{2v\left(2t+v\right)},
\end{aligned}
$$

$$
\begin{aligned}
\Pi_i^* &= \frac{\begin{array}{c}\left(\begin{array}{c} 48av^4 + 144tv^4 + 6a^2t^3 - 24a^2v^3 + 240t^2v^3 + 96t^3v^2 + \\ +9a^2t^2v - 8atv^3 - 32at^3v - 80at^2v^2 - 18a^2tv^2 \end{array}\right) \times \\ \times \left(\begin{array}{c} 48av^4 + 48tv^4 - 14a^2t^3 - 24a^2v^3 + 80t^2v^3 + 32t^3v^2 + \\ +120atv^3 + 32at^3v + 112at^2v^2 - 62a^2tv^2 - 53a^2t^2v \end{array}\right)\end{array}}{128cv^2\left(2t+3v\right)^2\left(2t+v\right)^4}, \\
U^* &= \frac{46tv^3 - 8t^3v - a^2t^2 - 4a^2v^2 + 24t^2v^2 + 12v^4 - 16atv^2 - 8at^2v - 4a^2tv}{8v\left(2t+v\right)^2}, \\
CS^* &= \frac{14tv^3 - 16av^3 - 8t^3v + a^2t^2 + 4a^2v^2 - 8t^2v^2 + 12v^4 - 24atv^2 - 8at^2v + 4a^2tv}{8v\left(2t+v\right)^2}, \\
W^* &= \frac{\begin{array}{c}\left(\begin{array}{c} 48av^4 - 276tv^4 + 32t^4v - 10a^2t^3 - 248t^2v^3 - 16t^3v^2 - 72v^5 + \\ +216atv^3 + 64at^3v + 224at^2v^2 - 22a^2tv^2 - 31a^2t^2v \end{array}\right) \times \\ \times \left(\begin{array}{c} 14a^2t^3 + 24a^2v^3 - 48av^4 - 48tv^4 + 62a^2tv^2 + \\ +53a^2t^2v - 80t^2v^3 - 32t^3v^2 - 120atv^3 - 32at^3v - 112at^2v^2 \end{array}\right)\end{array}}{64cv^2\left(2t+3v\right)^2\left(2t+v\right)^4}
\end{aligned}
$$

Retail price efficiency requires

$$
\begin{aligned}
p_i^* &= \frac{1}{4t+2v}\left(at + 2av + 4tv\right) = 0 = p_i^{opt} \\
&\Leftrightarrow \; a^{\text{efficient price}} = -\frac{4tv}{t+2v} < 0.
\end{aligned}
$$

Investment efficiency requires

$$
I_i^* = I_i^{opt},
$$

which is impossible to achieve under the fixed access price approach, since

$$
\begin{aligned}
\max_a I_i^* &= \frac{\left(t+2v\right)\left(48tv^2 + 32t^2v + 24v^3 - a\left(25tv + 14t^2 + 12v^2\right)\right)a + 16\left(t+v\right)\left(2t+3v\right)tv^2}{8cv\left(2t+3v\right)\left(2t+v\right)^2} \\
FOC &: \; -\frac{1}{4}\left(t+2v\right)\frac{14at^2 + 12av^2 - 24tv^2 - 16t^2v - 12v^3 + 25atv}{cv\left(2t+3v\right)\left(2t+v\right)^2} = 0 \\
&\Leftrightarrow \; a^{invest} = 4v\frac{6tv + 4t^2 + 3v^2}{25tv + 14t^2 + 12v^2}, \\
SOC &: \; -\frac{1}{4}\left(t+2v\right)\frac{25tv + 14t^2 + 12v^2}{cv\left(2t+v\right)^2\left(2t+3v\right)} < 0,
\end{aligned}
$$

and

$$
\begin{aligned}
I_i^*\left(a^{invest}\right) - I_i^{opt} &= 2v\frac{45tv^2 + 39t^2v + 11t^3 + 18v^3}{c\left(2t+3v\right)\left(25tv + 14t^2 + 12v^2\right)} - \frac{1}{4c}\left(6v - t\right) \\
&= -\frac{\left(6v^2 - 2t^2 + 3tv\right)\left(27tv + 14t^2 + 12v^2\right)}{4c\left(2t+3v\right)\left(25tv + 14t^2 + 12v^2\right)} < 0
\end{aligned}
$$

provided that $v > t$ by assumption.

STAGE I: REGULATORY REGIME CHOICE

Suppose that the regulator maximizes the social welfare $\max_{a\leq\bar{a}} W^*$, where $\bar{a} \equiv \left(2v^2 - tv - 2t^2\right) / \left(t + 2v\right)$. For $v/100 = t = 1$, the benevolent regulator solves

$$\max_{a\leq 99} W = \frac{\left(\begin{array}{c} -274\,707\,690\,327a^4 + 6233\,692\,256\,326\,080a^3 - 2155\,515\,780\,984\,125\,760a^2 + \\ +182\,788\,113\,311\,743\,488\,000a + 182\,486\,915\,829\,708\,800\,000 \end{array}\right)}{315\,910\,856\,706\,048\,000c}$$

$$FOC \quad : \quad -67\frac{\left(\begin{array}{c} 5361\,979\,554\,686\,880a - 23\,260\,045\,732\,560a^2 + \\ +1366\,704\,927a^3 - 227\,348\,399\,641\,472\,000 \end{array}\right)}{26\,325\,904\,725\,504\,000c} = 0$$

$$\Leftrightarrow a = 16786,\ a = 177.21,\ a = 55.921.$$

$$SOC|_{a=55.921} : -67\frac{-5168\,899\,051\,680 \times 55.921 + 455\,568\,309 \times 55.921^2 + 595\,775\,506\,076\,320}{2925\,100\,525\,056\,000c} < 0.$$

The regulator's choice is $a^* = 55.921$ and in equilibrium

$$\begin{aligned}
I_i^* &= \frac{80.787}{c} < \frac{149.75}{c} = I_i^{opt},\ p_i^* = 57.059 > 0 = p_i^{opt}, \\
z_i^* &= 0.2147 < 0.5 = z_i^{opt},\ q_i^* = 0.7147 < 1 = q_i^{opt}, \\
Q^* &= 1.4294 < 2 = Q^{opt},\ \Pi_i^* = \frac{3325.8}{c}, \\
U^* &= 133.47 < 149.75 = U^{opt},\ CS^* = 51.91,\ W^* = \frac{15039}{c} < \frac{22425}{c} = W^{opt}.
\end{aligned}$$

6.1.4 The access price indexation rule

Let the access price charged by operator i, per subscriber of operator j using i's infrastructure, be defined by $a_i \equiv xI_i - yI_j$, where (x, y) is the pair of regulatory parameters.

STAGE III: RETAIL PRICE COMPETITION UNDER INDEXATION

Operator 0's problem,

$$\begin{aligned}
\max_{p_0} \Pi_0 &= I \times p_0 q_0 + a_0 q_1 I_0 - a_1 q_0 I_1 - cI_0^2/2 \\
FOC &: \frac{\left(\begin{array}{c} 2tvI_0 + 2tvI_1 + tI_1 a_1 + vI_0 a_0 + vI_1 a_1 + \\ -2tI_0 p_0 - 2tI_1 p_0 - 2vI_0 p_0 + vI_0 p_1 - 2vI_1 p_0 + vI_1 p_1 \end{array}\right)}{2tv} = 0 \\
SOC &: -\left(I_0 + I_1\right)\frac{t+v}{tv} < 0.
\end{aligned}$$

Operator 1's problem,

$$
\begin{aligned}
\max_{p_1} \Pi_1 &= I \times p_1 q_1 + a_1 q_0 I_1 - a_0 q_1 I_0 - c I_1^2/2 \\
FOC &: \frac{\left(\begin{array}{c} 2tvI_0 + 2tvI_1 + tI_0 a_0 + vI_0 a_0 + vI_1 a_1 + \\ -2tI_0 p_1 - 2tI_1 p_1 + vI_0 p_0 - 2vI_0 p_1 + vI_1 p_0 - 2vI_1 p_1 \end{array}\right)}{2tv} = 0 \\
SOC &: -\left(I_0 + I_1\right) \frac{t+v}{tv} < 0.
\end{aligned}
$$

In equilibrium,

$$
p_i^{**} = \frac{v\left(3ta_i + 3va_i + 6tv + 4t^2\right) I_i + \left(6tv^2 + 4t^2 v + 2t^2 a_j + 3v^2 a_j + 4tva_j\right) I_j}{(2t+v)(2t+3v)(I_i + I_j)}
$$

and

$$
\left\{\begin{array}{l}
x_i^{**} = \frac{(2t+3v+a_i)I_i + (2t+3v-a_j)I_j}{2(I_i+I_j)(2t+3v)} \\
z_i^{**} = \frac{v\left(2tv+3v^2-3ta_i-3va_i\right)I_i + \left(2tv^2-2t^2a_j-3v^2a_j-4tva_j+3v^3\right)I_j}{2v(2t+v)(2t+3v)(I_i+I_j)} \\
q_i^{**} = \frac{v\left(10tv+4t^2+6v^2-ta_i-2va_i\right)I_i + 2(t+v)\left(2tv+3v^2-ta_j-2va_j\right)I_j}{2v(2t+v)(2t+3v)(I_i+I_j)}
\end{array}\right. .
$$

STAGE II: INVESTMENTS UNDER INDEXATION

$$
\begin{aligned}
\max_{I_0} \Pi_0^{**} &= I \times p_0^{**} q_0^{**} + a_0 q_1^{**} I_0 - a_1 q_0^{**} I_1 - c I_0^2/2 \\
FOC &: \frac{d\Pi_0^{**}}{dI_0} = \frac{\partial \Pi_0^{**}}{\partial I_0} + \sum_{i=0}^{1} \frac{\partial \Pi_0^{**}}{\partial a_i} \frac{\partial a_i}{\partial I_0} = 0.
\end{aligned}
$$

Assuming investment symmetry in equilibrium

$$
I_0 = I_1,
$$

thus,

$$
a_0 = a_1 = a = (x-y) I_i, \; i = 0, 1.
$$

Plugging $I_0 = I_1$ and $a_0 = a_1$ into the operator 0's FOC we reach

$$
\frac{\left(\begin{array}{c}
3(t+2v)(y-x)\left(10t^2 x - 2t^2 y + 10v^2 x - 6v^2 y + 19tvx - 7tvy\right) I_0^2 + \\
-8v \left(\begin{array}{c} 8ct^3 + 3cv^3 - 8t^3 x - 12v^3 x + 6v^3 y + 14ctv^2 + \\ +20ct^2 v - 30tv^2 x - 28t^2 vx + 7tv^2 y + 2t^2 vy \end{array}\right) I_0 + \\
+16tv^2 (t+v)(2t+3v)
\end{array}\right)}{8v(2t+3v)(2t+v)^2} = 0,
$$

while the SOC, in the equilibrium, has to hold the following inequality,

$$-\frac{\left(\begin{array}{c} 8v\left(2t+3v\right)\left(8ct^3+3cv^3-8t^3x-12v^3x+14ctv^2+20ct^2v-30tv^2x-28t^2vx\right)+ \\ +\left(t+2v\right)\left(\begin{array}{c} 132t^3x^2+4t^3y^2+162v^3x^2+18v^3y^2-120t^3xy-180v^3xy+ \\ +433tv^2x^2+404t^2vx^2+33tv^2y^2+20t^2vy^2-462tv^2xy-408t^2vxy \end{array}\right)I_0 \end{array}\right)}{8v\left(2t+v\right)^2\left(2t+3v\right)^2}<0.$$

STAGE I: REGULATORY REGIME UNDER INDEXATION

Suppose that the regulator maximizes the social welfare under the indexation approach, i.e., solves the following problem

$$\max_{x,y} W \equiv \left(I_0+I_1\right)U-c\left(I_0^2+I_1^2\right)/2$$
$$=\left(I_0+I_1\right)\left(v\left(1+z_0+z_1\right)-\frac{t\left(x_0^2+\left(1-x_0\right)^2\right)+2v\left(z_0^2+z_1^2\right)}{2}\right)-c\left(\frac{I_0^2}{2}+\frac{I_1^2}{2}\right)$$

subject to

$$x_i^{**}\left(I_i,I_j,x,y\right)=x_i^{**}\text{ (Stage III)}$$
$$z_i^{**}\left(I_i,I_j,x,y\right)=z_i^{**}\text{ (Stage III)}$$
$$q_i^{**}\left(I_i,I_j,x,y\right)=q_i^{**}\text{ (Stage III)}$$
$$p_i^{**}\left(I_i,I_j,x,y\right)=p_i^{**}\text{ (Stage III)}$$
$$d\Pi_i^{**}/dI_i=0\text{ (Stage II)}$$
$$d^2\Pi_i^{**}/dI_i^2\leq 0\text{ (Stage II)}$$
$$\Pi_i^{**}\geq 0\text{ (PC).}$$

The regulator's problem under the indexation approach can be rewritten as

$$\max_{x,y} W = I_0\frac{\left(\begin{array}{c} 2v\left(23tv^2+12t^2v-4t^3+6v^3\right)+ \\ -4v\left(4ct^2+cv^2+2t^2\left(x-y\right)+4tv\left(c+x-y\right)\right)I_0+ \\ -\left(x-y\right)^2\left(t+2v\right)^2I_0^2 \end{array}\right)}{4v\left(2t+v\right)^2}\text{ subject to}$$
$$\left\{\begin{array}{c} I_0\frac{16tv^2(v+t)-2v\left(4ct(t+v)+cv^2+v(2t+4v)(y-x)\right)I_0-(x-y)^2(t+2v)^2I_0^2}{4v(2t+v)^2}\geq 0\text{ (PC)} \\ d\Pi_i^{**}/dI_i=0\ \text{ (Op. FOC)} \\ d^2\Pi_i^{**}/dI_i^2<0\text{ (Op. SOC)} \end{array}\right.$$

Follows a numerical example for $v=100$, $t=1$, $c=1$. Since PC binds by Lemma 1, we solve repetitively the system below for different levels of I_i^{**} to find the welfare-maximizing investment under the indexation rule. The output of the numerical simulation is in Table A.1.

$$
\left\{
\begin{array}{c}
W = I_0 \frac{2v\left(23tv^2+12t^2v-4t^3+6v^3\right)-4v\left(4ct^2+cv^2+2t^2(x-y)+4tv(c+x-y)\right)I_0-(x-y)^2(t+2v)^2I_0^2}{4v(2t+v)^2} \\
\Pi_i = I_0 \frac{16tv^2(v+t)-2v\left(4ct(t+v)+cv^2+v(2t+4v)(y-x)\right)I_0-(x-y)^2(t+2v)^2I_0^2}{4v(2t+v)^2} = 0 \\
FOC : \frac{\left(\begin{array}{c}
-3\left(t+2v\right)\left(x-y\right)\left(10t^2x-2t^2y+10v^2x-6v^2y+19tvx-7tvy\right)I_0^2+ \\
-8v\left(\begin{array}{c} 8ct^3+3cv^3-8t^3x-12v^3x+6v^3y+14ctv^2+ \\ +20ct^2v-30tv^2x-28t^2vx+7tv^2y+2t^2vy \end{array}\right)I_0+ \\
+16tv^2\left(t+v\right)\left(2t+3v\right)
\end{array}\right)}{8v(2t+3v)(2t+v)^2} = 0 \\
SOC : -\frac{1}{8}\frac{\left(\begin{array}{c}
8v\left(2t+3v\right)\left(\begin{array}{c} 8ct^3+3cv^3-8t^3x-12v^3x+ \\ +14ctv^2+20ct^2v-30tv^2x-28t^2vx \end{array}\right)+ \\
+\left(t+2v\right)\left(\begin{array}{c} 132t^3x^2+4t^3y^2+162v^3x^2+ \\ +18v^3y^2-120t^3xy-180v^3xy+ \\ +433tv^2x^2+404t^2vx^2+33tv^2y^2+ \\ +20t^2vy^2-462tv^2xy-408t^2vxy \end{array}\right)I_0
\end{array}\right)}{v(2t+v)^2(2t+3v)^2} < 0 \\
p_0 = \frac{1}{4t+2v}\left(4tv+txI_0-tyI_0+2vxI_0-2vyI_0\right) \\
a_0 = xI_0 - yI_1,\; a_1 = xI_1 - yI_0 \\
v = 100,\; t = 1,\; c = 1 \\
I_1 = I_0 = I_0^{**}
\end{array}
\right.
$$

Table A.1: Numerical simulation for $v/100 = t = \tau = c = 1$.

I_i^{**}	x	y	p_i^{**}	W^{**}
$149.75 = I_i^{opt}$	$0.392\,32$	$6.758\,1 \times 10^{-2}$	49.875	18700
140	$0.374\,77$	$6.111\,2 \times 10^{-2}$	45.228	19466
130	$0.359\,62$	$5.608\,7 \times 10^{-2}$	40.839	19867
126	$0.354\,20$	$5.445\,8 \times 10^{-2}$	39.172	19928
125	$0.352\,89$	$5.408\,1 \times 10^{-2}$	38.763	19934
$124 = I_i^{SB}$	$0.351\,61$	$5.371\,7 \times 10^{-2}$	38.356	$19938 = \max W^{**} = W^{SB}$
123	$0.350\,34$	$5.336\,5 \times 10^{-2}$	37.952	19938
122	$0.349\,09$	$5.302\,4 \times 10^{-2}$	37.55	19935
120	$0.346\,65$	$5.237\,8 \times 10^{-2}$	36.754	19919
110	$0.335\,44$	$4.981\,5 \times 10^{-2}$	32.918	19653
100	$0.325\,69$	4.833×10^{-2}	29.289	19092
90	$0.317\,21$	$4.795\,2 \times 10^{-2}$	25.838	18254
$80.787 = I_i^{*}$	$0.310\,43$	$4.869\,6 \times 10^{-2}$	$22.795 < 57.059 = p_i^{*}$	$17249 > 15039 = W^{*}$
80	$0.309\,90$	$4.881\,4 \times 10^{-2}$	22.54	17154
70	$0.303\,72$	$5.120\,1 \times 10^{-2}$	19.377	15802

A welfare-maximizing regulator would implement the access price rule

$$a_i = 0.351\,61 I_i - 0.05371\,7 I_j,$$

inducing to the following equilibrium,

$$\begin{aligned}
I_i^{SB} &= 124 > 80.\,787 = I_i^*, \\
a_i^{SB} &= 36.\,938 < 55.\,921 = a^*,\ p_i^{SB} = 38.\,356 < 57.\,059 = p_i^*, \\
z_i^{SB} &= 0.308\,22 > 0.214\,7 = z_i^*,\ q_i^{SB} = 0.808\,22 > 0.714\,7 = q_i^*, \\
Q^{SB} &= 1.616\,4 > 1.429\,4 = Q^*,\ U^{SB} = 142.\,39 > 133.47 = U^*, \\
\Pi_i^{SB} &= 0 < 3325.8 = \Pi_i^*,\ W^{SB} = 19938 > 15039 = W^*.
\end{aligned}$$

6.1.5 The regulatory holidays case

Monopoly offering both brands: two hinterlands served. In a city monopolized by operator i unable to price discriminate, the demand function faced by the monopolist is defined by

$$q_i = \begin{cases} 2\left(\frac{v-p_i}{2v} + \frac{v-p_i}{t}\right) & \text{if } v \geq p_i > v - \frac{t}{2} \\ \frac{v-p_i}{v} + 1 & \text{if } p_i \leq v - \frac{t}{2} \end{cases}.$$

Operator i chooses p_i and I_i solving the following maximization problem

$$\max_{p_i, I_i} \Pi_i^{mon} = I_i p_i q_i - c I_i^2/2.$$

Suppose that $v > t$ and $p_i^{mon} = v - \frac{t}{2}$ and, check now if the monopolist has incentive to deviate the price by an ε. If the monopolist increases the price by ε will get

$$\Pi_i^{mon} = I_i \left(v - \frac{t}{2} + \varepsilon\right) 2 \left(\frac{v - \left(v - \frac{t}{2} + \varepsilon\right)}{2v} + \frac{v - \left(v - \frac{t}{2} + \varepsilon\right)}{t}\right) - c\frac{I_i^2}{2}$$

where

$$\frac{d\Pi_i^{mon}}{d\varepsilon} = I_i \left(t + 2v\right) \frac{t - v - 2\varepsilon}{tv} < 0, \text{ for } v > t,$$

therefore, the monopolist does not have incentive to increase the price above $p_i = v - \frac{t}{2}$ given that $v > t$. If the monopolist decreases the price by ε will get

$$\Pi_i^{mon} = I_i \left(v - \frac{t}{2} - \varepsilon\right) \left(\frac{v - \left(v - \frac{t}{2} - \varepsilon\right)}{v} + 1\right) - c\frac{I_i^2}{2}$$

where

$$\frac{d\Pi_i^{mon}}{d\varepsilon} = -I_i \frac{t+2\varepsilon}{v} < 0.$$

Therefore, the monopolist does not have incentive to decrease the price below $p_i = v - \frac{t}{2}$. We conclude that $p_i^{mon} = v - \frac{t}{2}$ and $q_i^{mon} = 1 + \frac{t}{2v}$.

The monopolist chooses the investment by solving

$$\begin{aligned} \max_{I_i} \Pi_i^{mon} &= I_i \left(v - \frac{t}{2}\right)\left(1 + \frac{t}{2v}\right) - cI_i^2/2 \\ FOC &: -\frac{1}{4}\frac{t^2 - 4v^2 + 4cvI_i}{v} = 0 \Leftrightarrow I_i^{mon} = \frac{4v^2 - t^2}{4cv}, \end{aligned}$$

and obtains a total profit of

$$\Pi_i^{mon} = \frac{(2v-t)^2 (2v+t)^2}{32cv^2}.$$

With regard to social welfare, in the monopoly equilibrium with both hinterlands served we have

$$\begin{aligned} x_i &= \frac{1}{2},\ z_i = \frac{v - p_i}{2v} = \frac{t}{4v} \\ U^{mon} &= v(x_0 + x_1 + z_0 + z_1) - \frac{t(x_0^2 + x_1^2) + 2v(z_0^2 + z_1^2)}{2} \\ &= v\left(1 + \frac{t}{2v}\right) - \frac{\frac{t}{2} + 4v\left(\frac{t}{4v}\right)^2}{2} = \frac{(4v-t)(t+2v)}{8v} \\ W^{mon} &\equiv \left(I_0^{mon} + I_1^{mon}\right) U^{mon} - c\left(\frac{(I_0^{mon})^2}{2} + \frac{(I_1^{mon})^2}{2}\right) \\ &= \frac{4v^2 - t^2}{2cv}\frac{(4v-t)(t+2v)}{8v} - c\left(\frac{\left(\frac{4v^2-t^2}{4cv}\right)^2}{2} + \frac{\left(\frac{4v^2-t^2}{4cv}\right)^2}{2}\right) \\ &= \frac{(2v-t)(t+2v)^2}{8cv}. \end{aligned}$$

In the numerical example, $v/100 = t = c = 1$, the outcome is

$$\begin{aligned} I_i^{mon} &= 100 < 124 = I_i^{SB},\ p_i^{mon} = 99.5 > 38.356 = p_i^{SB}, \\ Q^{mon} &= 1.005 < 1.6164 = Q^{SB},\ U^{mon} = 100.25 < 142.39 = U^{SB}, \\ \Pi_i^{mon} &= 4999.8 > 0 = \Pi_i^{SB},\ W^{mon} = 10050 < 19938 = W^{SB}. \end{aligned}$$

Monopoly offering one brand: one hinterland served. Assume that $v > 2t$. In this case the demand function faced by the monopolist is defined by

$$q_i = \begin{cases} \frac{v-p_i}{t}\left(1+\frac{t}{2v}\right) & \text{if } v \geq p_i > v-t \\ 1+\frac{v-p_i}{2v} & \text{if } p_i \leq v-t \end{cases}.$$

Operator i chooses p_i and I_i solving the following maximization problem

$$\max_{p_i, I_i} \Pi_i^{mon} = I_i p_i q_i - cI_i^2/2.$$

Suppose that $p_i^{mon} = v - t$ and check now if the monopolist has incentive to deviate the price by an ε. If the monopolist increases the price by ε will get

$$\Pi_i^{mon} = I_i\left(v-t+\varepsilon\right)\left(\frac{t-\varepsilon}{t}\left(1+\frac{t}{2v}\right)\right) - c\frac{I_i^2}{2}$$

where

$$\frac{d\Pi_i^{mon}}{d\varepsilon} = \frac{1}{2}I_i\left(t+2v\right)\frac{2t-v-2\varepsilon}{tv} < 0, \text{ for } v > 2t,$$

therefore, the monopolist does not have incentive to increase the price above $p_i = v - t$ given that $v > 2t$. If the monopolist decreases the price by ε will get

$$\Pi_i^{mon} = I_i\left(v-t-\varepsilon\right)\left(\frac{t+\varepsilon}{2v}+1\right) - c\frac{I_i^2}{2}$$

where

$$\frac{d\Pi_i^{mon}}{d\varepsilon} = -I_i\frac{2t+v+2\varepsilon}{2v} < 0,$$

therefore, the monopolist does not have incentive to decrease the price below $p_i = v - t$. We conclude that $p_i^{mon} = v - t$ and $q_i^{mon} = 1 + \frac{t}{2v}$.

The monopolist chooses the investment by solving

$$\begin{aligned} \max_{I_i} \Pi_i^{mon} &= I_i\left(v-t\right)\left(1+\frac{t}{2v}\right) - c\frac{I_i^2}{2} \\ FOC &: -\frac{tv+t^2-2v^2+2cvI_i}{2v} = 0 \Leftrightarrow I_i^{mon} = \frac{\left(v-t\right)\left(t+2v\right)}{2cv}, \end{aligned}$$

and obtains a total profit of

$$\Pi_i^{mon} = \frac{\left(t+2v\right)^2\left(v-t\right)^2}{8cv^2}.$$

With regard to social welfare, in the monopoly equilibrium with one hinterland served we have

$$
\begin{aligned}
x_i &= 1,\, x_j = 0,\, z_i = \frac{t}{2v},\, z_j = 0, \\
U^{mon} &= v\left(x_0 + x_1 + z_0 + z_1\right) - \frac{t\left(x_0^2 + x_1^2\right) + 2v\left(z_0^2 + z_1^2\right)}{2} \\
&= v\left(1 + \frac{t}{2v}\right) - \frac{t + 2v\left(\frac{t}{2v}\right)^2}{2} = \frac{(2v - t)(t + 2v)}{4v} \\
W^{mon} &\equiv \left(I_0^{mon} + I_1^{mon}\right) U^{mon} - c\left(\left(I_0^{mon}\right)^2 + \left(I_1^{mon}\right)^2\right)/2 \\
&= \frac{2v^2 - tv - t^2}{cv} \frac{(2v - t)(t + 2v)}{4v} - c\left(\frac{2v^2 - tv - t^2}{2cv}\right)^2 = \frac{(v - t)(t + 2v)^2}{4cv}.
\end{aligned}
$$

In the numerical example, $v/100 = t = c = 1$, the outcome is

$$
\begin{aligned}
I_i^{mon} &= 99.495 < 124 = I_i^{SB},\, p_i^{mon} = 99 > 38.356 = p_i^{SB}, \\
Q^{mon} &= 1.005 < 1.6164 = Q^{SB},\, U^{mon} = 99.998 < 142.39 = U^{SB}, \\
\Pi_i^{mon} &= 4949.6 > 0 = \Pi_i^{SB},\, W^{mon} = 9999.2 < 19938 = W^{SB}.
\end{aligned}
$$

6.2 Proofs

Proof of Proposition 1 (i) The socially efficient investment is defined by (10). In order to implement investment efficiency with the fixed access price rule is required that I_i^*, defined by (14), satisfies $I_i^* = I_i^{opt}$. However, $\max_a I_i^* < I_i^{opt}$ as is shown below.

$$
\begin{aligned}
\max_a I_i^* &= \frac{(t + 2v)\begin{pmatrix} 48tv^2 + 32t^2v + 24v^3 + \\ -a\left(25tv + 14t^2 + 12v^2\right) \end{pmatrix} a + 16(t + v)(2t + 3v)tv^2}{8cv(2t + 3v)(2t + v)^2} \\
FOC &: -\frac{1}{4}(t + 2v)\frac{14at^2 + 12av^2 - 24tv^2 - 16t^2v - 12v^3 + 25atv}{cv(2t + 3v)(2t + v)^2} = 0 \\
&\Leftrightarrow a^{invest} = 4v\frac{6tv + 4t^2 + 3v^2}{25tv + 14t^2 + 12v^2}, \\
SOC &: -\frac{1}{4}(t + 2v)\frac{25tv + 14t^2 + 12v^2}{cv(2t + v)^2(2t + 3v)} < 0,
\end{aligned}
$$

and

$$
\begin{aligned}
I_i^*\left(a^{invest}\right) - I_i^{opt} &= 2v\frac{45tv^2 + 39t^2v + 11t^3 + 18v^3}{c(2t + 3v)\left(25tv + 14t^2 + 12v^2\right)} - \frac{1}{4c}(6v - t) \\
&= -\frac{\left(3tv - 2t^2 + 6v^2\right)\left(27tv + 14t^2 + 12v^2\right)}{4c(2t + 3v)\left(25tv + 14t^2 + 12v^2\right)} < 0
\end{aligned}
$$

provided that $v > t$ by assumption.

(ii) Retail price efficiency requires

$$\begin{aligned} p_i^* &= \frac{1}{4t+2v}\left(at+2av+4tv\right)=0=p_i^{opt} \\ \Leftrightarrow\ a^{\text{efficient price}} &= -\frac{4tv}{t+2v}<0.\ \square \end{aligned}$$

Proof of Theorem 1 (i) Network i chooses the investment level by, given $\left(p_i^*, p_j^*\right)$,

$$\frac{\partial \Pi_i^*}{\partial I_i}\left(I_i^*\right)=\frac{\partial \Pi_i}{\partial I_i}+\frac{\partial \Pi_i^*}{\partial p_j}\frac{\partial p_j^*}{\partial I_i}=0.$$

In the first-best, according to (b), the regulator solves

$$\frac{\partial W}{\partial I_i}\left(I_i^{opt}\right)=\frac{\partial \Pi_i}{\partial I_i}+\frac{\partial \Pi_j}{\partial I_i}+CS=0, \tag{28}$$

where $CS \geq 0$ by definition and

$$\frac{\partial \Pi_j}{\partial I_i}=\left(p_j-a\right)q_j \geq 0 \text{ since } p_j \geq a.$$

By assumption (a) Π_i is strictly concave in I_i, and $CS+\frac{\partial \Pi_j}{\partial I_i} > \frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j^*}{\partial I_i}$ by (b). Hence, $I_i^{opt} > I_i^*$. (ii) The marginal cost of serving fibre subscribers is zero, thus $p_i^{opt}=0$. Networks choose retail prices by solving

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_i} &= 0 \Leftrightarrow \left(I_i+I_j\right)\left(q_i+p_i^*\frac{\partial q_i}{\partial p_i}\right)+a\left(\frac{\partial q_j}{\partial p_i}I_i-I_j\frac{\partial q_i}{\partial p_i}\right)=0 \\ \Leftrightarrow\ p_i^* &= \frac{a\left(\frac{\partial q_j}{\partial p_i}I_i-I_j\frac{\partial q_i}{\partial p_i}\right)+q_i\left(I_i+I_j\right)}{-\left(\partial q_i/\partial p_i\right)\left(I_i+I_j\right)}. \end{aligned}$$

Therefore, in order to get

$$\begin{aligned} p_i^* &= p_i^{opt} \Leftrightarrow \frac{a\left(\frac{\partial q_j}{\partial p_i}I_i-I_j\frac{\partial q_i}{\partial p_i}\right)+q_i\left(I_i+I_j\right)}{-\left(\partial q_i/\partial p_i\right)\left(I_i+I_j\right)}=0 \\ &\Leftrightarrow\ a=-\frac{q_i\left(I_i+I_j\right)}{\frac{\partial q_j}{\partial p_i}I_i-I_j\frac{\partial q_i}{\partial p_i}}<0, \end{aligned}$$

since $\partial q_j/\partial p_i > 0$ and $\partial q_i/\partial p_i < 0$ by assumption (c). $\square$

Proof of Lemma 1 The Lagrangean function of the regulator's problem is

$$L=W\left(x,y,I_0\right)+\lambda_1\left[\Pi_i\left(x,y,I_0\right)\right]+\lambda_2\left[S\left(x,y,I_0\right)\right]+\lambda_3\left[F\left(x,y,I_0\right)\right],$$

where $\Pi_i\left(x,y,I_0\right)$, $S\left(x,y,I_0\right)$ and $F\left(x,y,I_0\right)$ denote the network i's profit, the second and the first order conditions, respectively. The optimality conditions from the regulator's problem are

$$\begin{cases} L'_x \leq 0,\ xL'_x = 0 \\ L'_y \leq 0,\ yL'_y = 0 \\ L'_{\lambda_1} \geq 0,\ \lambda_1 L'_{\lambda_1} = 0,\ \lambda_1 \geq 0 \\ L'_{\lambda_2} < 0,\ \lambda_2 L'_{\lambda_2} = 0,\ \lambda_2 = 0 \\ L'_{\lambda_3} = 0,\ \lambda_3 L'_{\lambda_3} = 0 \end{cases}.$$

Showing that the participation constraint is binding is equivalent to showing that the respective Lagrange multiplier, λ_1, is different from zero. Suppose that $x \neq 0$ and $y \neq 0$, thus, $L'_x = 0$ and $L'_y = 0$. Solving the system of simultaneous equations

$$\begin{cases} L'_x = W'_x + \lambda_1 \Pi'_x + \lambda_3 F'_x = 0 \\ L'_y = W'_y + \lambda_1 \Pi'_y + \lambda_3 F'_y = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1^* = \frac{F'_x W'_y - F'_y W'_x}{\Pi'_x F'_y - \Pi'_y F'_x} \\ \lambda_3^* = \frac{\Pi'_y W'_x - \Pi'_x W'_y}{\Pi'_x F'_y - \Pi'_y F'_x} \end{cases} \tag{29}$$

for non-negative access prices, $a_i \geq 0$, i.e., $x \geq y$, we have

$$F'_x W'_y - F'_y W'_x = \frac{I_0^3 (t+2v)^2 \left(4tv + (t+2v)(x-y) I_0\right)\left(8tv + 6v^2 - 3I_0 (t+v)(x-y)\right)}{4v^2 (2t+3v)(2t+v)^3} \neq 0,$$

since $4tv + (t+2v)(x-y) I_0 > 0$ and $8tv + 6v^2 - 3I_0 (t+v)(x-y) \neq 0 \Leftrightarrow I_0 \neq \frac{8t+6v}{3(t+v)(x-y)} v$. To see that $I_0 \neq \frac{8t+6v}{3(t+v)(x-y)} v$, suppose by contradiction that $I_0 = \frac{8t+6v}{3(t+v)(x-y)} v$ and plug the expression into the first-order condition $F(x, y, I_0) = 0$. We get then

$$F\left(x, y, \frac{8t+6v}{3(t+v)(x-y)} v\right) = -\frac{\begin{pmatrix} 4cv(4t+3v)(2t+v)(t+v) + \\ +v\left(28tv^2 + 17t^2 v + 2t^3 + 12v^3\right)(x-y) \end{pmatrix}}{6(2t+v)(t+v)^2 (x-y)} < 0,$$

hence the FOC is not satisfied and $I_0 \neq \frac{8t+6v}{3(t+v)(x-y)} v$ must hold. Provided that $F'_x W'_y - F'_y W'_x \neq 0$ and $\lambda_1 \geq 0$, we conclude that $\lambda_1^* > 0$. Therefore, the participation constraint binds. $\square$

Proof of Proposition 2 (i) Under fixed access the regulator sets $a_i = a^*$. We can show that for any given access price $a^* > 0$ networks invest more under indexation than under fixed access. Under fixed access networks choose the investment level by $\partial \Pi_i^* / \partial I_i = 0$, since $a_i = a^*$ is fixed and $\partial a_i / \partial I_i = 0$, while under indexation networks choose the investment level by condition

$$\frac{\partial \Pi_i^{**}}{\partial I_i} + \sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i} = 0, \tag{30}$$

where $\partial a_k/\partial I_i = \begin{cases} x & \text{if } i = k \\ -y & \text{if } i \neq k \end{cases}$. We can show that $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$ provided that

$$\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} = I_0\left(t+2v\right)\frac{\begin{pmatrix} 12v^3x - 9av^2x + 3av^2y - 2atvy + 24tv^2x - 16atvx + \\ +16t^2vx - 8at^2x + 16t^2vy - 4at^2y + 16tv^2y \end{pmatrix}}{4v\left(2t+3v\right)\left(2t+v\right)^2}$$

and $12v^3x > 9av^2x$, $3av^2y > 2atvy$, $24tv^2x > 16tvax$, $16t^2vx > 8t^2ax$, $16t^2vy > 4t^2ay$, for $(x,y) \in \mathbb{R}_+^2$, $v > a > 0$ and $v > t$ by assumption. By (30) and the fact that $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$ then $\partial \Pi_i^{**}/\partial I_i < 0$. For $a_i = a^*$, (12) and (21) are identical, thus $\partial \Pi_i^{*}/\partial I_i = \partial \Pi_i^{**}/\partial I_i < 0$ and by the (SOC) concavity of the profit function with respect to I_i, i.e., $\partial^2 \Pi_i^{*}/\partial I_i^2 < 0$, we conclude that $I_i^{**} > I_i^{*}$.

(ii) The total mass of subscribers in a representative city is determined by (8), thus the mass of subscribers will expand if retail prices decrease. Retail prices will decrease if a_i decreases. Suppose that the regulator chooses to implement $a_i = a^* - \varepsilon$, where $\varepsilon > 0$. We can show that this is compatible with $I_i^{**} > I_i^{*}$ for ε sufficiently small. Replacing a_i by $a^* - \varepsilon$ in $\partial \Pi_i^{**}/\partial I_i$ we can show that

$$\frac{\partial \Pi_i^{**}}{\partial I_i}\left(a^* - \varepsilon\right) = \frac{\partial \Pi_i^{**}}{\partial I_i}\left(a^*\right) - \frac{\begin{pmatrix} \left(14t^3 + 24v^3 + 62tv^2 + 53t^2v\right)\left(\varepsilon - 2a\right) + \\ + \left(48v^4 + 120tv^3 + 32t^3v + 112t^2v^2\right) \end{pmatrix}}{8v\left(2t+3v\right)\left(2t+v\right)^2}\varepsilon$$

and in the limit

$$\lim_{\varepsilon \to 0} \frac{\left(14t^3 + 24v^3 + 62tv^2 + 53t^2v\right)\left(\varepsilon - 2a\right) + \left(48v^4 + 120tv^3 + 32t^3v + 112t^2v^2\right)}{8v\left(2t+3v\right)\left(2t+v\right)^2}\varepsilon = 0. \tag{31}$$

Hence, by continuity of the expression in (31) there exists $\varepsilon > 0$ such that

$$-\frac{\begin{pmatrix} \left(14t^3 + 24v^3 + 62tv^2 + 53t^2v\right)\left(\varepsilon - 2a\right) + \\ + \left(48v^4 + 120tv^3 + 32t^3v + 112t^2v^2\right) \end{pmatrix}}{8v\left(2t+3v\right)\left(2t+v\right)^2}\varepsilon + \sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$$

provided that $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$ as shown in (i). Condition (30) implies that $\partial \Pi_i^{**}\left(a^*\right)/\partial I_i < 0$, which combined with the concavity of Π_i^{**} with respect to I_i results in $I_i^{**}\left(a^* - \varepsilon\right) > I_i^{*}\left(a^*\right)$ for $\varepsilon > 0$ sufficiently small.

(iii) The social welfare variation can be approximately given by Taylor's first-order approximation

$$\Delta W \approx \frac{\partial W}{\partial U}\Delta U + \sum_{i=0}^{1} \frac{\partial W}{\partial I_i}\Delta I_i. \tag{32}$$

From (9) we have that $\partial W/\partial U = I_0 + I_1 > 0$, $\partial W/\partial I_i = U - cI_i$, while $\Delta U > 0$ and $\Delta I_i > 0$ come as consequence of (i) and (ii). In the first-best $\partial W/\partial I_i = U - cI_i = 0$, thus in the second-best $U - cI_i \geq 0$. Otherwise the regulator could increase the social welfare level by providing less incentives on investments. $\square$

Proof of Theorem 2 (i) We can show that for any given access price $a_i\left(I_i^{**}, I_j^{**}\right) = a_j\left(I_j^{**}, I_i^{**}\right) = a^* > 0$ networks invest more under indexation than under a fixed access. Under fixed access networks choose the investment level by condition $\partial \Pi_i^*/\partial I_i = 0$ while under indexation networks choose investments by

$$\frac{\partial \Pi_i^{**}}{\partial I_i} + \sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i} = 0 \tag{33}$$

where $\partial \Pi_i^{**}/\partial I_i = \partial \Pi_i^*/\partial I_i$ if $a_i\left(I_i^{**}, I_j^{**}\right) = a^*$. Note that by (a)

$$\begin{aligned}
\frac{\partial \Pi_i^{**}}{\partial a_i} &= \underbrace{I_i q_j^{**}}_{\geq 0} + \underbrace{\left.\frac{\partial \Pi_i}{\partial p_i}\right|_{p_i^{**}=p_i}}_{\substack{=0,\text{ by FOC}\\ \text{in price stage}}} \cdot \frac{\partial p_i^{**}}{\partial a_i} + \underbrace{\left.\frac{\partial \Pi_i}{\partial p_j}\right|_{p_j^{**}=p_j}}_{\geq 0,\text{ by (b)}} \cdot \underbrace{\frac{\partial p_j^{**}}{\partial a_i}}_{\geq 0} \geq 0, \text{ where} \\
\frac{\partial p_j^{**}}{\partial a_i} &= -\frac{\partial^2 \Pi_j/\partial p_j \partial a_i}{\partial^2 \Pi_j/\partial p_j^2} \geq 0 \text{ since } \partial^2 \Pi_j/\partial p_j^2 < 0 \text{ by (c) and} \\
\partial^2 \Pi_j/\partial p_j \partial a_i &= -I_i . \partial q_j/\partial p_j \geq 0 \text{ by (d).}
\end{aligned}$$

Given that, by (d), Π_i is twice differentiable, $\partial \Pi_i^{**}/\partial a_j$ must be finite. Hence, the regulator can choose $\partial a_i/\partial I_i$ and $\partial a_j/\partial I_i$ such that $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$. For example, setting $\partial a_i/\partial I_i > 0$ and $\partial a_j/\partial I_i = 0$ would suffice. Since $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k}\frac{\partial a_k}{\partial I_i} > 0$, thus $\partial \Pi_i^{**}/\partial I_i = \partial \Pi_i^*/\partial I_i < 0$ by (33). Due to sufficient convexity of $C(I_i)$, Π_i^* is concave in I_i (SOC in the investment stage). Therefore, the investment solution I_i^{**} defined by (33) must be higher than I_i^* defined by $\partial \Pi_i^*/\partial I_i = 0$.

(ii) Suppose that the regulator intends to implement an access price $a_i\left(I_i^{**}, I_j^{**}\right) = a^* - \varepsilon$, for $\varepsilon > 0$. We show that this is compatible with having $I_i^{**}\left(a^* - \varepsilon\right) > I_i^*\left(a^*\right)$ for ε sufficiently small, while equilibrium prices decrease with ε. Regarding retail prices,

$$\begin{aligned}
\partial p_j^{**}/\partial a_i &\geq 0 \text{ as shown in (i),} \\
\frac{\partial p_i^{**}}{\partial a_i} &= -\frac{\partial^2 \Pi_i/\partial p_i \partial a_i}{\partial^2 \Pi_i/\partial p_i^2} \geq 0 \text{ since } \partial^2 \Pi_i/\partial p_i^2 < 0 \text{ by (c) and} \\
\partial^2 \Pi_i/\partial p_i \partial a_i &= I_i . \partial q_j/\partial p_i \geq 0 \text{ by (d).}
\end{aligned}$$

Regarding investments, replacing a^* by $a^* - \varepsilon$ in $\partial \Pi_i^*/\partial I_i$ and taking the first-order approximation we get

$$\frac{\partial \Pi_i^*}{\partial I_i}\left(a^* - \varepsilon\right) \approx \frac{\partial \Pi_i^*}{\partial I_i}\left(a^*\right) - \varepsilon \frac{\partial^2 \Pi_i^*}{\partial I_i \partial a}\left(a^*\right) \tag{34}$$

where $\frac{\partial^2 \Pi_i^*}{\partial I_i \partial a}(a^*)$ is finite, due to twice differentiability of Π_i^*, and independent of ε. Thus,

$$\lim_{\varepsilon \to 0} \varepsilon \frac{\partial^2 \Pi_i^*}{\partial I_i \partial a^*}(a^*) = 0.$$

By the continuity of the expression in (34) and the fact that $\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i} > 0$ as shown in (i), there exists an $\varepsilon > 0$ such that

$$\sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i} - \varepsilon \frac{\partial^2 \Pi_i^*}{\partial I_i \partial a}(a^*) > 0.$$

By (33) we get that $\partial \Pi_i^{**}(a^*)/\partial I_i = \partial \Pi_i^*(a^*)/\partial I_i < 0$ and conclude that, by concavity of Π_i^* with respect to I_i, $I_i^{**}(a^* - \varepsilon) > I_i^*(a^*)$ for $\varepsilon > 0$ sufficiently small.

(iii) The social welfare variation can be approximately computed by Taylor's first-order approximation

$$\Delta W \approx \frac{\partial W}{\partial U} \Delta U + \sum_{i=0}^{1} \frac{\partial W}{\partial I_i} \Delta I_i,$$

where $\partial W/\partial U = I_i + I_j > 0$, $\partial W/\partial I_i = U - C'(I_i)$, while $\Delta I_i > 0$ and $\Delta U > 0$ are consequence of the results in (i), and (ii) together with assumption (f), respectively. In the first-best $\partial W/\partial I_i = U - C'(I_i) = 0$. Thus, in equilibrium it must be the case that $U - C'(I_i) \geq 0$. Otherwise the regulator could increase social welfare by providing less incentives on investments. □

Proof of Proposition 3 For $I_i = I_i^{opt} = (6v - t)/(4c)$ to be implemented with the indexation rule a regulatory regime (x, y) has to pass three tests: (i) the network FOC in (23) (ii) the SOC whose signal is defined by (24) and (iii) profits, defined by (20), have to be non-negative, otherwise networks exit the market. Therefore, the efficient investment can be implemented if there exists a regulatory policy (x, y) that satisfies

$$\text{(i)} \left\{ \begin{array}{c} 3(t+2v)(y-x)\left(10t^2x - 2t^2y + 10v^2x - 6v^2y + 19tvx - 7tvy\right)\left(\frac{6v-t}{4c}\right)^2 + \\ -8v \left(\begin{array}{c} 8ct^3 + 3cv^3 - 8t^3x - 12v^3x + 6v^3y + 14ctv^2 + \\ +20ct^2v - 30tv^2x - 28t^2vx + 7tv^2y + 2t^2vy \end{array} \right) \frac{6v-t}{4c} + \\ +16tv^2(t+v)(2t+3v) \end{array} \right\} = 0,$$

$$\text{(ii)} \left\{ \begin{array}{c} 8v(2t+3v) \left(\begin{array}{c} 8ct^3 + 3cv^3 - 8t^3x - 12v^3x + \\ +14ctv^2 + 20ct^2v - 30tv^2x - 28t^2vx \end{array} \right) + \\ +(t+2v) \left(\begin{array}{c} 132t^3x^2 + 4t^3y^2 + 162v^3x^2 + 18v^3y^2 - 120t^3xy + \\ -180v^3xy + 433tv^2x^2 + 404t^2vx^2 + 33tv^2y^2 + \\ +20t^2vy^2 - 462tv^2xy - 408t^2vxy \end{array} \right) \frac{6v-t}{4c} \end{array} \right\} > 0,$$

$$\text{(iii)}\ 16tv^2(v+t) - 2v \left(\begin{array}{c} 4ct(t+v) + cv^2 + \\ +v(2t+4v)(y-x) \end{array} \right) \frac{6v-t}{4c} - (x-y)^2(t+2v)^2 \left(\frac{6v-t}{4c}\right)^2 \geq 0.$$

If the participation constraint is active and parameters (v, t, c) satisfy the SOC whose signal is defined by (24), i.e.,

$$S \equiv c \frac{\left(\begin{pmatrix} -444\,320t^{10}v - 4272\,480t^{9}v^{2} - 13\,002\,620t^{8}v^{3} + \\ -12\,144t^{11} - 139\,968v^{11} + 16\,472\,160tv^{10} + \\ +103\,878\,864t^{2}v^{9} + 269\,586\,480t^{3}v^{8} + 373\,756\,596t^{4}v^{7} + \\ +289\,781\,298t^{5}v^{6} + 111\,303\,761t^{6}v^{5} + 3797\,783t^{7}v^{4} \end{pmatrix} + \\ -\frac{\begin{pmatrix} 16v\left(1056tv^{4} + 1617t^{2}v^{3} + 1028t^{3}v^{2} + 216t^{4}v + 252v^{5} - 8t^{5}\right) + \\ -\left(4\left(t+2v\right)\begin{pmatrix} 1092tv^{4} + 412t^{4}v + 891t^{2}v^{3} + \\ +563t^{3}v^{2} + 132t^{5} + 468v^{5} \end{pmatrix}\right)\sqrt{\frac{2v}{t+2v}} \end{pmatrix}}{51tv^{2}+54v^{3}-4t^{2}v-9t^{3}} \times \\ \times \left(t+v\right)\begin{pmatrix} 442\,440tv^{6} - 12\,212t^{6}v + 638\,472t^{2}v^{5} + \\ +384\,866t^{3}v^{4} + 40\,005t^{4}v^{3} - 47\,334t^{5}v^{2} + 600t^{7} + 112\,752v^{7} \end{pmatrix} \right)}{16\left(6v-t\right)\begin{pmatrix} 51tv^{2} - 4t^{2}v + \\ -9t^{3} + 54v^{3} \end{pmatrix}\begin{pmatrix} 1092tv^{4} + 412t^{4}v + 891t^{2}v^{3} + \\ +563t^{3}v^{2} + 132t^{5} + 468v^{5} \end{pmatrix}\left(2t+v\right)^{2}} < 0 \tag{35}$$

together with $v > t > 0$ and $c > 0$ by assumption, then the regulatory solution (x, y) will be defined by the zero-profit condition and (23). Taking the limit of S

$$\lim_{t \to 0} S = -\frac{11}{36}c < 0$$

clearly satisfies condition (35). Therefore, if service differentiation, t, is sufficiently small, the regulatory regime (x, y) defined by the zero-profit condition and (23), by continuity of S, will implement the efficient level of investment. $\square$

Proof of Theorem 3 With an access pricing rule depending on investments, network i chooses the investment level by

$$\frac{\partial \Pi_i^{**}}{\partial I_i} + \sum_{k=0}^{1} \frac{\partial \Pi_i^{**}}{\partial a_k} \frac{\partial a_k}{\partial I_i} = 0,$$

where $\partial \Pi_i^{**}/\partial I_i = \partial \Pi_i/\partial I_i + (\partial \Pi_i/\partial p_j)\left(\partial p_j^{**}/\partial I_i\right)$. Therefore, i's investment choice is defined by

$$\frac{\partial \Pi_i}{\partial I_i} + \frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j^{**}}{\partial I_i} + q_j I_i \frac{\partial a_i}{\partial I_i} - q_i I_j \frac{\partial a_j}{\partial I_i} = 0.$$

Recall from (28) that the first-best investment is defined by $\frac{\partial \Pi_i}{\partial I_i} + \frac{\partial \Pi_j}{\partial I_i} + CS = 0$. Hence, if the regulator defines a_i such that $\frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j^{**}}{\partial I_i} + q_j I_i \frac{\partial a_i}{\partial I_i} - q_i I_j \frac{\partial a_j}{\partial I_i} = \frac{\partial \Pi_j}{\partial I_i} + CS$ is fulfilled and profits are non-negative, the first-best investment level will be implemented. By (a), for a sufficiently convex cost function, the SOC of the investment problem is satisfied. $\square$

Proof of Proposition 4 (i) Suppose that both hinterlands are served and the regulator, using indexation, intends to implement the retail price $p_i^{**} = v - t/2 - \varepsilon_p < p_i^{mon}$ and the investment level $I_i^{**} = \frac{4v^2-t^2}{4cv} + \varepsilon_I > I_i^{mon}$, where $\varepsilon_p, \varepsilon_I > 0$. This proof consists in verifying if it is possible to find a regulatory regime (x, y) such that networks have non-negative profits, (23) and (24) are satisfied for some $\varepsilon_p > 0$ and $\varepsilon_I > 0$.

By (21) and investment symmetry, retail prices in equilibrium follow $p_i^{**} = \frac{(t+2v)a_i+4tv}{2(2t+v)}$. In order to implement a retail price $p_i^{**} = v - \frac{t}{2} - \varepsilon_p$, the access price must satisfy

$$a_i = (x-y)\left(I_i^{mon} + \varepsilon_I\right) = \frac{2v^2 - tv - 2t^2}{t+2v} - \varepsilon_a$$

where $\varepsilon_a \equiv \frac{4t+2v}{t+2v}\varepsilon_p$. Moreover, the network choice regarding the investment level has to suffice (23). Solving the system of simultaneous equations in order to (x, y)

$$\begin{cases} (x-y)\left(\frac{4v^2-t^2}{4cv} + \varepsilon_I\right) = \frac{2v^2-tv-2t^2}{t+2v} - \varepsilon_a \\ \begin{pmatrix} 3(t+2v)(y-x)\left(10t^2x - 2t^2y + 10v^2x - 6v^2y + 19tvx - 7tvy\right)\left(\frac{4v^2-t^2}{4cv} + \varepsilon_I\right)^2 + \\ -8v \begin{pmatrix} 8ct^3 + 3cv^3 - 8t^3x - 12v^3x + 6v^3y + \\ +14ctv^2 + 20ct^2v - 30tv^2x - 28t^2vx + 7tv^2y + 2t^2vy \end{pmatrix} \left(\frac{4v^2-t^2}{4cv} + \varepsilon_I\right) + \\ +16tv^2(t+v)(2t+3v) \end{pmatrix} = 0 \end{cases}$$

we get

$$\Leftrightarrow \begin{cases} x^* = \dfrac{cv(2t+3v)\begin{pmatrix} 3t^2\varepsilon_a^2 + 12v^2\varepsilon_a^2 + 12tv^3 + 4t^3v + 12t^3\varepsilon_a - 8v^3\varepsilon_a + 9t^2v^2 + \\ +4t^4 + 4v^4 + 12tv\varepsilon_a^2 + 8tv^2\varepsilon_a + 30t^2v\varepsilon_a + 8cv^3\varepsilon_I + 32ctv^2\varepsilon_I + 32ct^2v\varepsilon_I \end{pmatrix}}{(2t+v)(4v^2+4cv\varepsilon_I-t^2)(19tv^2+17t^2v+3t^2\varepsilon_a+6v^2\varepsilon_a+6t^3+6v^3+9tv\varepsilon_a)} \\ y^* = \dfrac{cv\begin{pmatrix} 30t^4\varepsilon_a^2 + 120v^4\varepsilon_a^2 - 124tv^5 + 404t^5v + 120t^5\varepsilon_a - 48v^5\varepsilon_a + \\ -122t^2v^4 + 259t^3v^3 + 574t^4v^2 + 104t^6 - 24v^6 + 348tv^3\varepsilon_a^2 + 748t^2v^3\varepsilon_a + \\ +177t^3v\varepsilon_a^2 + 1042t^3v^2\varepsilon_a + 378t^2v^2\varepsilon_a^2 + 120tv^4\varepsilon_a + 592t^4v\varepsilon_a + \\ +48cv^5\varepsilon_I + 432ct^2v^3\varepsilon_I + 288ct^3v^2\varepsilon_I + 248ctv^4\varepsilon_I + 64ct^4v\varepsilon_I \end{pmatrix}}{(2t+v)(t+2v)(4v^2+4cv\varepsilon_I-t^2)(19tv^2+17t^2v+3t^2\varepsilon_a+6v^2\varepsilon_a+6t^3+6v^3+9tv\varepsilon_a)} \end{cases} . \tag{36}$$

Plugging the previous regulatory regime (x^*, y^*) and $I_i^{**} = \frac{4v^2-t^2}{4cv} + \varepsilon_I$ into the SOC whose signal is defined by (24) and taking the limit for $(\varepsilon_I, \varepsilon_a) \to (0, 0)$ we get

$$\lim_{\varepsilon_a \to 0}\lim_{\varepsilon_I \to 0} \frac{S}{8v(2t+v)^2(2t+3v)^2} = \frac{c\begin{pmatrix} 2548t^7v - 5984tv^7 - 9136t^2v^6 - 6512t^3v^5 + \\ +604t^4v^4 + 5938t^5v^3 + 5687t^6v^2 + 476t^8 - 1728v^8 \end{pmatrix}}{8(2v-t)(t+2v)(7tv+3t^2+6v^2)^2(2t+v)^2} < 0,$$

since $2548t^7v < 9136t^2v^6$, $604t^4v^4 < 1728v^8$, $5938t^5v^3 + 476t^8 < 6512t^3v^5$, $5687t^6v^2 < 5984tv^7$, provided that $v > t > 0$ by assumption. Hence, by continuity of the SOC we can assure that there exists $\varepsilon_p > 0$ and $\varepsilon_I > 0$ sufficiently small such that (x^*, y^*) defined by (36) can decrease retail prices and increase the investment relatively to the regulatory holidays regime. Furthermore, since $\Pi_i^{mon} > 0$ for $(p_i, I_i) = (p_i^{mon}, I_i^{mon})$, by continuity of the profit function, for $\varepsilon_p > 0$ and $\varepsilon_I > 0$ sufficiently small we can guarantee that profits are still non-negative with the implementation of (x^*, y^*).

With regard to social welfare,

$$W^{mon} \equiv \left(I_0^{mon} + I_1^{mon}\right) U^{mon} - c\left(\frac{\left(I_0^{mon}\right)^2}{2} + \frac{\left(I_1^{mon}\right)^2}{2}\right)$$

and

$$\begin{aligned} U^{mon} &= v\left(x_0 + x_1 + z_0 + z_1\right) - \frac{t\left(x_0^2 + x_1^2\right) + 2v\left(z_0^2 + z_1^2\right)}{2} \\ x_i &= \frac{1}{2},\ z_i = \frac{v - p_i}{2v} = \frac{t}{4v}. \end{aligned}$$

Taking the derivatives of welfare in order to investments and retail prices

$$\begin{aligned} \frac{\partial W^{mon}}{\partial I_i} &= U^{mon} - cI_i^{mon} = \frac{(4v - t)(t + 2v)}{8v} - c\frac{4v^2 - t^2}{4cv} = t\frac{t + 2v}{8v} > 0, \\ \frac{\partial W^{mon}}{\partial p_i} &= \left(I_0^{mon} + I_1^{mon}\right)\frac{\partial U^{mon}}{\partial p_i} = \frac{4v^2 - t^2}{2cv} \times \sum_{i=0}^{1} \frac{\partial U^{mon}}{\partial z_i}\frac{\partial z_i}{\partial p_i} \\ &= -\frac{4v^2 - t^2}{2cv} \times \frac{v - 2vz_i^{mon}}{v} = -\frac{(t + 2v)(2v - t)^2}{4cv^2} < 0, \end{aligned}$$

since $v > t > 0$ and $c > 0$. Therefore, for a sufficiently small increase in investments and a sufficiently small decrease in retail prices, the welfare level increases relatively to the regulatory holidays case.

(ii) Suppose that only one hinterland is served and that the regulator, using indexation, intends to implement the retail price $p_i^{**} = v - t - \varepsilon_p < p_i^{mon}$ and the investment level $I_i^{**} = \frac{2v^2 - tv - t^2}{2cv} + \varepsilon_I > I_i^{mon}$, where $\varepsilon_p, \varepsilon_I > 0$. In order to implement a retail price $p_i^{**} = v - t - \varepsilon_p$, the access price must satisfy

$$a_i = (x - y)\left(I_i^{mon} + \varepsilon_I\right) = \frac{2(v - 2t)(t + v)}{t + 2v} - \varepsilon_a$$

where $\varepsilon_a \equiv \frac{4t+2v}{t+2v}\varepsilon_p$. Solving the system of simultaneous equations in order to (x, y)

$$\left\{ \begin{array}{c} (x - y)\left(\frac{2v^2 - tv - t^2}{2cv} + \varepsilon_I\right) = \frac{2(v-2t)(t+v)}{t+2v} - \varepsilon_a \\ \left(\begin{array}{c} 3(t + 2v)(y - x)\left(10t^2x - 2t^2y + 10v^2x - 6v^2y + 19tvx - 7tvy\right)\left(\frac{2v^2 - tv - t^2}{2cv} + \varepsilon_I\right)^2 + \\ -8v\left(\begin{array}{c} 8ct^3 + 3cv^3 - 8t^3x - 12v^3x + 6v^3y + \\ +14ctv^2 + 20ct^2v - 30tv^2x - 28t^2vx + 7tv^2y + 2t^2vy \end{array} \right)\left(\frac{2v^2 - tv - t^2}{2cv} + \varepsilon_I\right) + \\ +16tv^2(t + v)(2t + 3v) \end{array} \right) = 0 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x^* = \dfrac{cv(2t+3v)\begin{pmatrix} 3t^2\varepsilon_a^2 + 12v^2\varepsilon_a^2 + 4tv^3 + 16t^3v + 24t^3\varepsilon_a - 8v^3\varepsilon_a - 8t^2v^2 + 32t^4 + \\ +4v^4 + 12tv\varepsilon_a^2 + 20tv^2\varepsilon_a + 60t^2v\varepsilon_a + 8cv^3\varepsilon_I + 32ctv^2\varepsilon_I + 32ct^2v\varepsilon_I \end{pmatrix}}{2(2t+v)(2v^2+2cv\varepsilon_I-tv-t^2)(22tv^2+26t^2v+3t^2\varepsilon_a+6v^2\varepsilon_a+12t^3+6v^3+9tv\varepsilon_a)} \\ y^* = \dfrac{cv\begin{pmatrix} 30t^4\varepsilon_a^2 + 120v^4\varepsilon_a^2 - 172tv^5 + 1472t^5v + 240t^5\varepsilon_a - 48v^5\varepsilon_a - 204t^2v^4 + \\ +640t^3v^3 + 1728t^4v^2 + 448t^6 - 24v^6 + 348tv^3\varepsilon_a^2 + 1276t^2v^3\varepsilon_a + \\ +177t^3v\varepsilon_a^2 + 1852t^3v^2\varepsilon_a + 378t^2v^2\varepsilon_a^2 + 240tv^4\varepsilon_a + 1120t^4v\varepsilon_a + \\ +48cv^5\varepsilon_I + 432ct^2v^3\varepsilon_I + 288ct^3v^2\varepsilon_I + 248ctv^4\varepsilon_I + 64ct^4v\varepsilon_I \end{pmatrix}}{2(2t+v)(t+2v)(2v^2+2cv\varepsilon_I-tv-t^2)(22tv^2+26t^2v+3t^2\varepsilon_a+6v^2\varepsilon_a+12t^3+6v^3+9tv\varepsilon_a)} \end{cases}.$$

Plugging the regulatory regime (x^*, y^*) and $I_i^{**} = \frac{2v^2-tv-t^2}{2cv} + \varepsilon_I$ into the SOC whose signal is defined by (24) and taking the limit for $(\varepsilon_I, \varepsilon_a) \to (0,0)$ we get

$$\lim_{\varepsilon_a\to 0}\lim_{\varepsilon_I\to 0} \frac{S}{8v\,(2t+v)^2\,(2t+3v)^2} = c\frac{\begin{pmatrix} 4128t^6v^2 - 108v^8 + 2808t^5v^3 - 338tv^7 - 629t^2v^6 + \\ +247t^4v^4 - 812t^3v^5 + 656t^8 + 2624t^7v \end{pmatrix}}{4\,(2t+v)^2\,(v-t)\,(t+2v)\,(5tv+3t^2+3v^2)^2} < 0$$

since $4128t^6v^2 < 108v^8$, $2808t^5v^3 < 338tv^7$, $247t^4v^4 < 629t^2v^6$, $2624t^7v + 656t^8 < 812t^3v^5$, given that $v > 2t > 0$. Hence, a linear access pricing rule depending on investments can decrease retail prices and increase investments as compared to the regulatory holidays regime.

With regard to social welfare, taking the derivatives in order to investments and retail prices,

$$\begin{aligned} \frac{\partial W^{mon}}{\partial I_i} &= U^{mon} - cI_i^{mon} = \frac{(2v-t)\,(t+2v)}{4v} - c\frac{2v^2-tv-t^2}{2cv} = t\frac{t+2v}{4v} > 0, \\ \frac{\partial W^{mon}}{\partial p_i} &= (I_0^{mon} + I_1^{mon})\,\frac{\partial U^{mon}}{\partial p_i} = \frac{(v-t)\,(t+2v)}{cv} \times \frac{\partial U^{mon}}{\partial z_i}\frac{\partial z_i}{\partial p_i} \\ &= -\frac{(v-t)\,(t+2v)}{cv} \times \frac{(v-2vz_i^{mon})}{2v} = -\frac{(t+2v)\,(v-t)^2}{2cv^2} < 0, \end{aligned}$$

since $v > t > 0$. Therefore, for a sufficiently small increase in investments and a sufficiently small decrease in retail prices, the welfare level increases relatively to the regulatory holidays case. □