NOVA SCHOOL OF BUSINESS AND ECONOMICS

Microeconomics
Fall 2011

Class #2: Cost Minimization. Profit Maximization.

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Classes: 40, 41

Question 1
Consider a firm that has a Cobb-Douglas production technology $q = K^{\alpha}L^{\alpha}$. The firm wishes to minimize the cost of producing q units of output and has access to perfectly competitive factor markets. Let r and w denote the prices of one unit of K and L, respectively.
a) Formalize the firm's problem. What are the variables and the parameters of the problem?
b) Find the conditional demand functions. Label them $L^*(w, r, q)$ and $K^*(w, r, q)$.
c) Find the cost function $C(w, r, q)$. What is its interpretation?
d) Find $\frac{\partial C}{\partial q}$, $\frac{\partial C}{\partial w}$, $\frac{\partial C}{\partial r}$ and interpret.
e) How does $\frac{dC}{dq}$ vary with α? Explain intuitively using the concept of "returns to scale".

Question 2
Consider a firm that produces holes with shovels (K) and people (L). Assume that people and shovels are perfect complements so that technology is given by:

$$q = \min\{K, L\}.$$

The firm wishes to minimize the cost of producing Y units of output and has access to perfectly competitive factor markets. Let r and w denote the prices of one unit of K and L, respectively.
a) What is the firm's cost minimization problem?
b) What is the optimality condition?
c) Find the conditional demand functions. Label them $L^*(w, r, q)$ and $K^*(w, r, q)$.
d) Find the cost function: $C(w, r, q)$. What is particular about the expression you obtained? Discuss the economic intuition of your result.

Question 3
Consider an industry where the total cost of each firm equals $C = 2 + 20q + 2q^2$.
a) Find the average total cost, the average variable cost and the average fixed cost.
b) Derive the supply function from one firm.
c) Assume that there are two firms in this industry and the demand is defined by $P = 30 - Q/4$. Find the short-run equilibrium of this market: price, quantity produced by each firm and total quantity traded.
d) Find the long-run free entry equilibrium of this industry: price, quantity produced by each firm, total quantity traded and number of firms.