**Microeconomic Theory II**
**PS1**

1. Let $p \in P$ be a simple lottery and let

$$U(p) = \sum_{j=1}^{n} p(x_j) \cdot \sqrt{x_j}$$

be a vNM-utility function. Here $x_j \in \mathbb{R}_+$, $j = 1, 2, ..., n$ is a given set of money prices.

   (a) Show that this utility function satisfies the independence axiom.

   (b) Consider now the utility function $V(p) = [U(p)]^2$. If you think that $V$ satisfies the independence axiom, prove it. If you think that V does not satisfy the independence axiom, give a counterexample.

   (c) Calculate a certainty equivalent if $U(p) = \frac{2\sqrt{x_1}+\sqrt{x_2}}{3}$.

2. A gambler with current wealth $Y$ has the chance to bet any amount on the occurrence of an event that she knows will occur with probability $p$. If she wagers $w$, she will receive $2w$ if the event occurs and 0 if it doesn't. She has a constant risk aversion coefficient utility $u(y) = -e^{-ry}$ with $r > 0$. What should be the gambler's bet?

3. Let $u_1(w)$ and $u_2(w)$ be two differentiable, increasing and concave Bernoulli utility functions on amounts of money, $w$. Consider the following properties:
   i) $-u_1''(w)/u_1'(w) > -u_2''(w)/u_2'(w)$ for all $w$.
   ii) $u_1(w) = F(u_2(w))$ for some increasing strictly concave function $F$.
   iii) $\pi_{u_1}(\tilde{\epsilon}) > \pi_{u_2}(\tilde{\epsilon})$ for all random variables $\tilde{\epsilon}$ with zero expected value, i.e. $E(\tilde{\epsilon}) = 0$. Note: $\pi_{u_i}(\tilde{\epsilon})$ denotes the risk premium for $u_i$, $i = 1, 2$, as a function of $\tilde{\epsilon}$.

   (a) Prove that properties i) and ii) are equivalent.

   (b) Prove that properties ii) and iii) are equivalent.

4. True or false?

   (a) If $G(x)$ first-order-stochastically dominates $F(x)$, the expected value of $x$ under $G$ is not smaller than the expected value of $x$ under $F$.

   (b) If the expected value of $x$ under $G$ is greater than the expected value of $x$ under $F$, then $G(x)$ first-order-stochastically dominates $F(x)$.

   (c) If $G(x)$ second-order-stochastically dominates $F(x)$ and they have the same mean, the variance of $x$ under $G$ is not greater than the variance of $x$ under $F$.

   (d) If $G$ and $F$ have the same mean and $G$ has a smaller variance than $F$, then $G$ second-order stochastically dominates $F$.

   For each statement, provide a proof or a counterexample.

5. Consider the two-player game represented below:

| | L | R |
|---|---|---|
| U | a,b | c,d |
| D | e,f | g,h |

If possible (otherwise, explain),

(a) Find examples of values for the parameters such that there is a rationalizable pure strategy for one of the players that is not part of a Nash equilibrium.

(b) Find examples of values for the parameters such that there are exactly three pure strategy Nash equilibria.

(c) Find examples of values for the parameters such that there is a unique pure strategy Nash equilibrium that is not an equilibrium in dominant strategies.

(d) Find examples of values for the parameters such that there is a unique mixed strategy Nash equilibrium in which each player plays either of his actions with probability 1/2.

(e) Find the ranges of values for the parameters such that there are exactly two pure strategy Nash equilibrium.

(f) A number of different values for the parameters $a$, $c$, $e$ and $g$ lead to the same best response correspondence for player 1; there are then some degrees of freedom in this problem: how many values do you really need to know in order to determine player 1's best response correspondence?

6. Consumers are uniformly distributed along a boardwalk that is 1 mile long and with unitary demand. Newspapers prices are regulated, so consumers go to the nearest vendor because they dislike walking (assume that at the regulated prices all consumers will purchase one newspaper even if they have to walk a full mile). If more than one vendor is at the same location, they split the business evenly. Suppose the demand is normalized to $N = 1$.

(a) Consider a game in which two newspaper vendors pick their locations simultaneously. Show that there is a unique pure strategy Nash equilibrium.

(b) Show that with three vendors, no pure strategy Nash equilibrium exists.

7. (Extra credit) Under the conditions in MWG Proposition 8.D.3, show that any 2-person symmetric game (where $S_1 = S_2$ and $u_i(s_i; s_j) = u_j(s_j; s_i)$) has a symmetric Nash equilibrium.