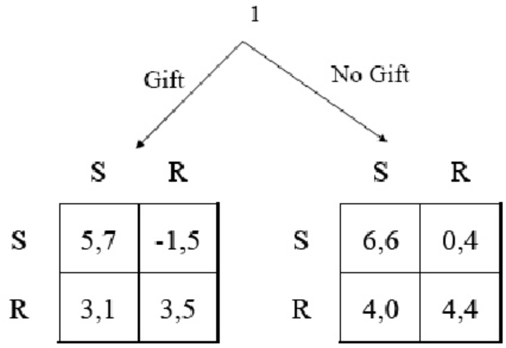
Microeconomic Theory II
PS2

1) Consider the following 2-player game where each player simultaneously decides whether to play R or S and the payoff matrix is

	S	R
S	$6, 6$	$0, 4$
R	$4, 0$	$4, 4$

Suppose that before the game, player 1 can offer a gift to player 2 which is worth 1 utile to player 2 and costs 1 utile to player 1. The players know whether the gift is given before the game, and all these are common knowledge. Hence, the game is as follows.



Find all subgame-perfect equilibria in pure strategies.

2) Consider the entry deterrence game, where an Entrant decides whether to enter the market; if he enters the Incumbent decides whether to Fight or Accommodate. Consider a game where Incumbent's payoff from the Fight is private information, the entry deterrence game is repeated twice and the discount rate is $\delta = 0.9$. The payoff vectors for the stage game are $(0, 2)$ if the Entrant does not enter, $(-1, a)$ if he enters and the Incumbent Fights; and $(1, 1)$ if he enters and the Incumbent accommodates, where the first entry in each parenthesis is the payoff for the entrant. Here, a can be either -1 or 2, and is privately known by the Incumbent. Entrant believes that $a = -1$ with probability $\pi = 0.9$; and everything described up to here is common knowledge. Find the perfect Bayesian Equilibrium.

3) Consider a "repeated" game, where a long-run firm plays à la Stackelberg against a short-run firm that is in the market only for one period, while the long-run firm remains in the market throughout the game. At each period t, first, the short run firm sets its quantity x_t; then, knowing x_t, the long-run firm sets its quantity y_t; and each sells his good at price $p_t = 1 - (x_t + y_t)$. The marginal costs are all 0. The short-run firm maximizes its profit, which incurs at t. The long-run firm maximizes the present value of its profit stream where the discount rate is $\delta = 0.99$. At the beginning of each period, the actions taken previously are all common knowledge.

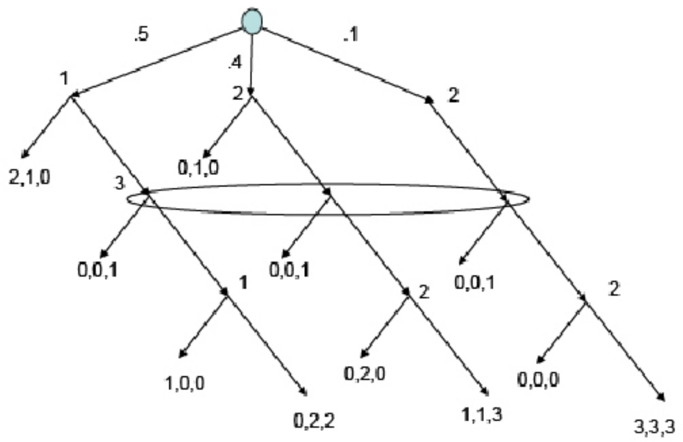
(a) What is the subgame perfect equilibrium if there are only finitely many dates, i.e., $t \in \{0, 1, ..., T\}$.

(b) Now consider the infinitely repeated game. Find a subgame perfect equilibrium, where $x_t = 1/4$ and $y_t = 1/2$ at each t on the path of equilibrium play, namely in the contingencies that happen with positive probability given the strategies.

(c) Can you find a subgame perfect equilibrium, where $x_t = y_t = 1/4$ for each t on the path of equilibrium play?

4) Consider a simultaneous game where two players invest in a project. Each player choose a non-negative investment level. If player i invests x_i and the other player j invests x_j, then the payoff of player i is $\theta_i x_i x_j - x_i^3$. Here, θ_i is privately known by partner i, and the other partner believes that θ_i is uniformly distributed on $[0, 1]$. All these are common knowledge. Find a symmetric Bayesian Nash equilibrium in which the investment of partner i is in the form of $x_i = a + b\sqrt{\theta_i}$.

5) Find a perfect Bayesian Nash equilibrium in the following game.



6) Recall the first-price sealed-bid auction example seen in class. Assume instead that the object has a "common value" of $\theta = x_1 + x_2$ i.e. the value of the object is the sum of the individual valuations x_1 and x_2 that are iid according to the $U[0, 1]$. Each buyer knows her valuation but not the other one's.

(a) Assuming linearity of the bids, what are the equilibrium bids?

(b) If you don't assume linearity, is that the only equilibrium? If not, explain what conditions must be met in equilibrium.