Microeconomic Theory II
PS 4

1. A firm faces a continuum of customer types, $\theta \in [0,1]$. Assume the types are distributed uniformly. A type-θ consumer gets utility $(\theta+1)x - \frac{1}{2}x^2$ from consuming x units of the firm's good. The firm wants to maximize its expected profits by selling x-packs for $p(x)$ (assume the firm can limit any given customer to a single transaction). For convenience assume the marginal cost of production is zero. What is $p(\cdot)$?

2. Consider a sealed bid *all-pay* auction with I symmetric buyers. Each buyer's valuation θ_i is independently drawn from the interval $[0,1]$ according to a uniform density i.e. $\theta_i \sim U[0,1]$.

The mechanism is as follows: Every buyer submits a non-negative bid, the highest bidder receives the good, and *every* buyer pays the seller the amount of her bid *regardless of whether she wins.*

(a) Show that it is a symmetric Bayesian Nash equilibrium for each bidder to bid according to $b(\theta_i) = \frac{I-1}{I}\theta_i^I$.

(b) Assume that this is the unique Bayesian Nash equilibrium of the game. Argue, using the revenue equivalence theorem, that the auction yields the same expected revenue as the sealed-bid second price auction.

(c) Assume now that $I=3$ and that θ_1 is uniformly distributed on $[0,10]$, and θ_2 and θ_3 are uniformly distributed on $[0,1]$. Design an optimal auction. Explain how you might implement it by allowing asymmetric reserve bids in one of the standard auctions (sealed bid first-price, Vickrey...).

(d) With $I=3$, suppose instead that it is known that bidder 1 has the highest valuation: say θ_1 is uniformly distributed on $[1,11]$, and θ_2 and θ_3 are still uniformly distributed on $[0,1]$. What does the optimal auction look like in this case?

3. Consider the Myerson-Satterthwaite framework in which both traders' valuations (s for the seller and b for the buyer) are uniformly distributed on [0,1].

(a) Suppose an English auction is used to allocate the good between the seller and the buyer. The price rises continuously until one of the traders drops out. If the seller drops out first, the buyer gets the good at the seller's drop-out price. Show that the seller's optimal strategy is to drop out at the price $1/2 + s/2$ and the buyer's optimal strategy is to drop out at the price b.

(b) Show that the extensive form in (a) implements the ex ante efficient mechanism that maximizes the seller's expected payoff (i.e., the welfare weights are 1 for the seller and 0 for the buyer). Is the mechanism ex post efficient?

4. Consider the following version of a regulated-monopoly problem.

The monopolist's revenue as a function of output q is $R(q) = q.p(q)$; assume that $R'(q) > 0$ for all q. The monopolist's cost function is $C(q,\theta) = \theta q$ where the constant marginal cost θ is private information for the monopolist. It is common knowledge that θ is drawn from a uniform distribution on the interval $\left[\underline{\theta}, \overline{\theta}\right]$, where $0 < \underline{\theta} < \overline{\theta}$.

The regulator *observes the firm's total cost C but does not observe output q* (nor θ). The regulator chooses a payment function $t : C \to \mathbb{R}$ and the firm receives $t(C)$. The monopolist's profit is then $\pi = R - C + t$.

Let $S = \int_0^q p(s)ds$ so that consumer surplus is $S - R$.

Using the revelation principle, the regulator will solve the problem by looking for menus of the form $\{(C(\theta), t(\theta)\}_{\theta \in [\underline{\theta}, \overline{\theta}]}$ that maximize her objective function $S - R + \alpha\pi - t = S - C - (1-\alpha)\pi$, where $0 < \alpha < 1$, subject to the incentive constraints and to the constraint that all types of firm make non-negative profits.

(a) What is the first best outcome in this model as a function of θ i.e. the outcome that the regulator would enforce if there was no private information and the regulator observed output as well as cost?

(b) Write down the first-order condition corresponding to the "local IC constraint" that types should prefer truthful reporting to reporting a type "slightly different" than their own. (Hint: note that *cost* and *not output* should now be used as the firm's choice variable, since we need that choice to be observable).

(c) What are the analogs of the "constant sign" conditions CS^+ and CS^- here? Which one implies that any implementable contract must have the realized total cost be a non-decreasing function of type?

(d) Rewrite the profit of the firm of type θ as the sum of the utility of the "worst type" and an integral.

(e) Assuming that the monotonicity constraint is slack, when might you expect that the optimum could involve producing so much output that price is less than marginal cost?

(f) Once the optimal menus $\{(C(\theta), t(\theta)\}_{\theta \in [\underline{\theta}, \overline{\theta}]}$ have been derived, what should a function $t : C \to \mathbb{R}$ look like? (Assume that the regulator may impose arbitrarily large fines, whenever that is necessary).

5. The Government of country A approved new legislation with respect to smoking in public places. In small restaurants, smoking will be allowed only if a new smoke-extraction technology is in place. The owner of restaurant B must now make a binary choice, denoted by x: x is 1 if the new technology is bought (and smoking is allowed) and 0 otherwise. Assume that the cost of the new technology is c and therefore $c(x) = cx$. Assume also that the owner of restaurant B cares only about the total welfare of its I customers (and about the cost of the technology), as long as she does not lose any money. Let t_i denote the (possibly negative) transfer from the owner to each agent and let each agent's utility function be given by $u_i(x, t_i, \theta_i) = v(x, \theta_i) + t_i = \theta_i \; x + t_i$ where θ_i is private information and has a distribution P_i on $[\underline{\theta}_i, \overline{\theta}_i]$ that is common knowledge.

(a) Assuming that no customer can stop going to this restaurant (it is the only one in the vicinity), describe the mechanism that achieves the Bayesian implementation of the efficient choice. Check that all the relevant constraints are satisfied.

(b) Assume that customers can choose to go to other restaurants, but the owner is now willing to lose money. Would it be possible to implement a mechanism that achieves dominant-strategy implementation of the efficient choice? If so, describe the mechanism. If not, explain why not.

6. FT 7.10