Industrial Organization

Problem Set #2 Solution

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1 Dominant Firm

(*6 points*)

$$\begin{aligned}
P &= 20-2Q^D \iff Q^D = 10-\frac{1}{2}P \\
MgC_A &= 9 \\
TC_i &= q_i\left(q_i+11\right) = q_i^2+11q_i,\ i=B,C. \\
MgC_i &= 2q_i+11,\ i=B,C. \\
\text{Price takers:} & \quad P = MgC_i \iff P = 2q_i+11 \iff q_i = \frac{1}{2}P-\frac{11}{2},\ i=B,C. \\
Q_{fringe}\left(P\right) &= 2\left(\frac{1}{2}P-\frac{11}{2}\right) = P-11 \iff P = Q_{fringe}+11 \qquad (\textit{1 point})
\end{aligned}$$

Firm A's residual demand,

$$Q_A = \begin{cases} 10-\frac{1}{2}P & P\leq 11 \\ \left[10-\frac{1}{2}P\right]-\left[P-11\right] & P>11 \end{cases} \iff Q_A = \begin{cases} 10-\frac{1}{2}P & P\leq 11 \\ 21-\frac{3}{2}P & P>11 \end{cases} \iff$$

$$\iff P = \begin{cases} 20-2Q_A & Q_A\geq 4.5 \\ 14-\frac{2}{3}Q_A & Q_A<4.5 \end{cases}.$$

Firm A's marginal revenue,

$$MgR_A = \begin{cases} 20-4Q_A & Q_A\geq 4.5 \\ 14-\frac{4}{3}Q_A & Q_A<4.5 \end{cases}.$$

Solving A's problem using the usual optimality condition:

$$MgR_A = MgC_A \iff 14-\frac{4}{3}Q_A = 9 \iff Q_A^* = 3.75 < 4.5 \quad (\textit{3 points})$$

and the equilibrium price is $P^* = 14-\frac{2}{3}\times 3.75 = 11.5.$ (*1 point*)

Given the price defined by the dominant firm, the competitive fringe in equilibrium produces

$$Q^*_{fringe}(11.5) = 11.5 - 11 = 0.5. \quad (\textit{1 point})$$

The total demand in equilibrium is given by,

$$Q^D(11.5) = Q^*_{fringe} + Q^*_A = 0.5 + 3.75 = 4.25.$$

2 Monopolistic competition

a) (*2 points*)

For any number of firms, n, NOVA's problem is,

$$\max_q \pi = \left(90 + \frac{20}{n} - 4q\right) q - q^2 - 414.05$$

By the f.o.c.:

$$90 + \frac{20}{n} - 8q - 2q = 0 \Longleftrightarrow q^* = 9 + \frac{2}{n} \text{ and } p^* = 90 + \frac{20}{n} - 4\left(9 + \frac{2}{n}\right) = 54 + \frac{12}{n}. \quad (1)$$

Checking the s.o.c.:

$$\frac{\partial^2 \pi}{\partial q^2} = -10 < 0. \text{ Thus, by concavity of } \pi, \text{ the solution from (1) is a maximum.}$$

Computing NOVA's profit we get,

$$\begin{aligned}\pi(n) &= \left(54 + \frac{12}{n}\right)\left(9 + \frac{2}{n}\right) - \left(9 + \frac{2}{n}\right)^2 - 414.05 \\ &= \left(9 + \frac{2}{n}\right)\left(45 + \frac{10}{n}\right) - 414.05 \\ &= 405 + \frac{180}{n} + \frac{20}{n^2} - 414.05 \quad (2)\end{aligned}$$

Therefore, in the short run, for $n = 4$,

$$\begin{aligned}q^*_{sr} &= 9 + \frac{2}{4} = 9.5 \quad (\textit{0.75 point}) \\ p^*_{sr} &= 54 + \frac{12}{4} = 57 \quad (\textit{0.75 points}) \\ \pi^*_{sr} &= 405 + \frac{180}{4} + \frac{20}{4^2} - 414.05 = 37.2 > 0. \quad (\textit{0.5 points})\end{aligned}$$

Since the profit is positive, it is expected that the number of firms operating in the market should increase till the zero profit condition is met.

b) (4 points)
Equating (2) to zero,

$$\pi(n) = 0 \Longleftrightarrow 405 + \frac{180}{n} + \frac{20}{n^2} - 414.05 = 0, \text{ Solution is : } \{n = -0.1105\}, \{n = 20.0\}$$

Since n must be non-negative, the only possible solution is $n_{LR} = 20$ firms. *(1 point)*

By q^* and p^* derived in (1) and $n_{LR} = 20$ it is obtained,

$$q^*_{LR} = 9 + \frac{2}{20} = 9.1 \text{ and } p^*_{LR} = 54 + \frac{12}{20} = 54.6 \quad \text{(1 point)}$$

If F increases, the impact in the number of firms in the long run is quantified by,

$$\begin{aligned} \frac{\partial n}{\partial F}(20, 414.05) &= -\frac{\partial \pi / \partial F}{\partial \pi / \partial n} = -\frac{-1}{-\frac{180}{n^2} - \frac{40}{n^3}}\bigg|_{(20,414.05)} = -\frac{n^3}{180n + 40}\bigg|_{(20,414.05)} \\ &= -\frac{200}{91} = -2.1978 < 0 \quad \text{(0.5 points)} \end{aligned}$$

where the first equality is assured by the Implicit Function Theorem and

$$\pi(n, F) = 405 + \frac{180}{n} + \frac{20}{n^2} - F = 0 \text{ in the long run.}$$

Alternatively, for those who are not familiar with the theorem, computing explicitly,

$$\begin{aligned} \pi(n, F) &= 405 + \frac{180}{n} + \frac{20}{n^2} - F = 0 \Longleftrightarrow (405 - F)\, n^2 + 180n + 20 = 0 \\ \Longleftrightarrow \quad & n = \frac{-180 - \sqrt{180^2 - 80(405 - F)}}{2(405 - F)}. \end{aligned}$$

Note: The solution with a plus before the root is ruled out since n is negative in that case.

Computing the derivative of n in order to F,

$$\frac{\partial n}{\partial F} = \frac{1}{4F^2 - 3240F + 656100}\left(-8\sqrt{5F} - 360\right) + \frac{2}{(2F - 810)\sqrt{\frac{1}{5}F}},$$

applying the derivative when $F = 414.05$,

$$\frac{\partial n}{\partial F}(414.05) = \frac{-8\sqrt{5 \times (414.05)} - 360}{4 \times 414.05^2 - 3240 \times 414.05 + 656100} + \frac{2}{(2 \times 414.05 - 810)\sqrt{\frac{1}{5} \times 414.05}} = -2.1978.$$

By equations in (1) it's easy to show that

$$\frac{\partial q^*}{\partial F} = \frac{\partial q^*}{\partial n}\frac{\partial n}{\partial F} = -\frac{2}{n^2} \cdot \frac{\partial n}{\partial F},$$

where the first equality holds according to the chain rule and $\frac{\partial n}{\partial F}$ is known from the previous computation. Hence, at the point $n = 20$, $\frac{\partial q^*}{\partial F} = -\frac{2}{20^2}(-2.1978) = 1.0989 \times 10^{-2} > 0$. *(0.5 points)*

Still using equations from (1),

$$\frac{\partial p^*}{\partial F} = \frac{\partial p^*}{\partial n}\frac{\partial n}{\partial F} = -\frac{12}{n^2}\frac{\partial n}{\partial F}.$$

Finding the derivative at the point where $n = 20$, we get $\frac{\partial p^*}{\partial F} = -\frac{12}{20^2} \times (-2.1978) = 6.5934 \times 10^{-2} > 0$. *(0.5 points)*

Intuitive conclusions. *(0.5 points)*

In the long run all firms are operating with zero profit. Thus, if the fixed cost increases, the profit becomes negative. Firms start leaving the market till the zero profit condition is met again, so the number of firms in the LR equilibrium is decreasing with F. The firms that still operate observe an expansion of the faced demand function which allows for an increase in revenue through higher prices and quantity sold. After all adjustments, the revenue increase should be exactly equal to the fixed-cost increase.

c) (3 points)

The short run consumer surplus (CS_{sr}),

$$CS_{sr} = n.\frac{\left(90 + \frac{20}{n} - p^*_{sr}\right) q^*_{sr}}{2} = 4.\frac{\left(90 + \frac{20}{4} - 57\right) 9.5}{2} = 722, \quad \textit{(1.5 points)}$$

i.e., consumers gain a surplus of $\frac{722}{4} = 180.5$ from each one of the products (and there are 4).

The long run consumer surplus (CS_{LR}),

$$CS_{LR} = n.\frac{\left(90 + \frac{20}{n} - p^*_{LR}\right) q^*_{LR}}{2} = 20.\frac{\left(90 + \frac{20}{20} - 54.6\right) 9.1}{2} = 3312.4, \quad \textit{(1.5 points)}$$

i.e., consumers gain a surplus of $\frac{3312.4}{20} = 165.62$ from each one of the products (and there are 20).

Notice that although consumers extract a lower surplus from each product (variety) in the long-run than in the short run equilibrium, the total generated surplus by the 20 firms exceeds by far the one generated by 4 firms.

d) (2 points)

Defining the Average Total Cost function (ATC),

$$ATC = \frac{TC}{q} = q + \frac{414.05}{q}.$$

Deriving the minimum of the ATC function,

$$\min_q ATC = q + \frac{414.05}{q},$$

by the f.o.c.

$$1-\frac{414.05}{q^2}=0 \iff q^{opt}=\sqrt{414.05}=20.348, \text{ the negative solution is ruled out.} \quad (\textit{1.5 points})$$

Checking the s.o.c.

$$\frac{\partial^2 ATC}{\partial q^2}=\frac{414.05\times 2}{q^3}=\frac{414.05\times 2}{20.348^3}=9.829\,2\times 10^{-2}>0, \text{ hence } ATC \text{ reaches a minimum at } q^{opt}=20.348.$$

However, $q^*_{sr}=9.5<20.348$, and $q^*_{LR}=9.1<20.348$. Concluding, firms operating in this market are producing non-optimaly either in the SR as in the LR. (*0.5 points*)

e) (*3 points*)
Suppose there are $n=n_0$ firms in the market, then the Lerner index expression is given by

$$\begin{aligned} L &= \frac{P-c}{P}=\frac{P(n_0)-2q(n_0)}{P(n_0)}=\frac{\left(54+\frac{12}{n_0}\right)-2\left(9+\frac{2}{n_0}\right)}{54+\frac{12}{n_0}}=\frac{36+\frac{8}{n_0}}{54+\frac{12}{n_0}} \\ &= \frac{36n_0+8}{54n_0+12}=\frac{36n_0+8}{(36n_0+8)\frac{3}{2}}=\frac{2}{3}. \quad (\textit{2 points}) \end{aligned}$$

$$\frac{P-c}{P}=\frac{2}{3}\iff P=3c,$$

i.e., the price is always 3 times the marginal cost independently of the number of firms in the market and obviously,

$$\lim_{n\to\infty}\frac{36+\frac{8}{n_0}}{54+\frac{12}{n_0}}=\frac{2}{3}.$$

Intuition for the result. Each time a firm enters the market it forces the incumbents to reduce the price and quantity produced and consequently also their marginal costs (given by $2q$).

Taking into account product differentiation and the fact that price and marginal cost are proportional for any n, the Lerner index remains constant which is equivalent to stating that firms' market power (mark-up) is independent of the number of competitors. (*1 point*)