Industrial Organization

Problem Set #3 Solution

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Part I

Cournot Competition

Exercise 1 (*7 points*)
a)

$$\begin{aligned} D(P) &= 100 - P \Longleftrightarrow P = 100 - Q \\ c_i &= 10, \text{ for } i = 1, 2. \end{aligned}$$

Firm 1's problem,

$$\max_{q_1} \pi_1 = (100 - (q_1 + q_2) - 10)\, q_1$$

By the f.o.c.,

$$90 - (2q_1 + q_2) = 0 \Longleftrightarrow q_1 = \frac{90 - q_2}{2}. \quad (\textit{0.5 points})$$

Checking the s.o.c.,

$$\frac{d^2\pi_1}{dq_1^2} = -2 < 0.$$

Firm 2's problem,

$$\max_{q_2} \{\pi_2 + \alpha q_2\} = (100 - (q_1 + q_2) - 10)\, q_2 + \alpha q_2$$

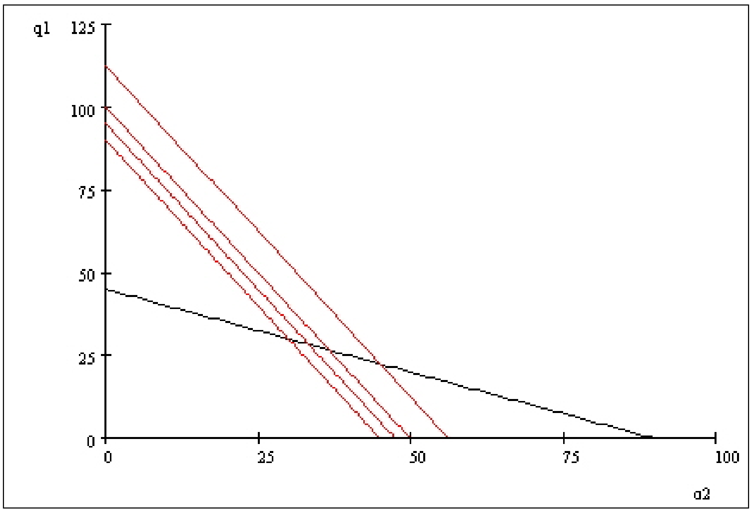
By the f.o.c.,

$$90 - (q_1 + 2q_2) + \alpha = 0 \Longleftrightarrow q_2 = \frac{90 + \alpha - q_1}{2}. \quad (\textit{0.5 points})$$

Checking the s.o.c.,

$$\frac{d^2(\pi_2 + \alpha q_2)}{dq_2^2} = -2 < 0.$$

(*1 point*) Representing graphically the best-response functions,



Black and red lines correspond to the best-response functions of firm 1 and 2, respectively. Notice that for higher levels of α the firm 2's reaction function goes up.

b) The equilibrium quantities are taken from the solution of the best-response system,

$$\begin{cases} q_1 = \frac{90-q_2}{2} \\ q_2 = \frac{90+\alpha-q_1}{2} \end{cases} \iff \begin{cases} q_1^* = \frac{90-\alpha}{3} \\ q_2^* = \frac{90+2\alpha}{3} \end{cases} . \quad (\textit{2 points}) \tag{1}$$

c)

$$\begin{aligned} Q &= q_1^* + q_2^* = \frac{90-\alpha}{3} + \frac{90+2\alpha}{3} = \frac{180+\alpha}{3} = 60 + \frac{\alpha}{3} \quad (\textit{0.5 points}) \\ P &= 100 - 60 - \frac{\alpha}{3} = 40 - \frac{\alpha}{3} \quad (\textit{0.5 points}) \end{aligned}$$

Firm 2's profit,

$$\max_{\alpha} \pi_2^* = \left(40 - \frac{\alpha}{3} - 10\right) \frac{90+2\alpha}{3}$$

$$\begin{aligned} FOC &: -\frac{1}{3} \times \frac{90+2\alpha}{3} + \left(40 - \frac{\alpha}{3} - 10\right)\frac{2}{3} = 0 \iff \alpha = \frac{45}{2} = 22.5. \quad (\textit{1 point}) \\ SOC &: -\frac{2}{9} - \frac{2}{9} = -\frac{4}{9} < 0, \text{ ensures that } \alpha^* = 22.5 \text{ is a maximizer of } \pi_2^*. \end{aligned}$$

Intuition (*1 point*): Recall that α is the weight that firm 2 attributes to level of production in its objective function. Hence, α can be understood as a measure of firm 2's aggressiveness. This is easy to see in the graph since an increase in α pushes firms 2's reaction function to north, i.e. for any q_1, firm 2 responds producing more.

Therefore, the fact that the α maximizer is positive implies that the best action that firm 2 can choose is to have an aggressive behaviour producing more than if it was a simple profit maximizer firm. Given this and taking into account the equilibrium expressions in (1), when firm 2 is more aggressive the best-response from firm 1 is to reduce q_1.

Another way of looking at α is that it allows firms 2 to choose freely the point it prefers in firm 1's best-response function, in fact, this is just a smart way of firm 2 to become be Stackelberg leader.

Exercise 2 (*6 points*)
Linear (inverse) demand functional form,

$$p = a - bq,$$

where a and b are positive parameters.
Let c_i and c_j be the marginal costs of firm i and j, respectively, such that,

$$c_i + c_j = c, \; i = 1, 2 \text{ and } i \neq j \tag{2}$$

and c is a positive constant.

Solving the firm's problem with Cournot competition,

$$\max_{q_i} \pi_i = (a - b(q_i + q_j) - c_i) q_i, \text{ for } i = 1, 2 \text{ and } i \neq j,$$

by the f.o.c.,

$$a - b(2q_i + q_j) - c_i = 0.$$

Verifying the s.o.c.,

$$\frac{d\pi_i}{dq_i} = -2b < 0.$$

In equilibrium we solve the best-response function system,

$$\begin{cases} a - b(2q_i + q_j) - c_i = 0 \\ a - b(2q_j + q_i) - c_j = 0 \end{cases} \iff \begin{cases} q_i^* = \frac{a - 2c_i + c_j}{3b} \\ q_j^* = \frac{a + c_i - 2c_j}{3b} \end{cases}.$$

Computing the C_1 index,

$$C_1 = \frac{q_i^*}{q_i^* + q_j^*} = \frac{\frac{a-2c_i+c_j}{3b}}{\frac{a-2c_i+c_j}{3b} + \frac{a+c_i-2c_j}{3b}} = \frac{a - 2c_i + c_j}{2a - c_i - c_j}, \text{ when } c_i < c_j \text{ (i.e. } q_i^* > q_j^*) \tag{3}$$

$$C_1 = \frac{q_j^*}{q_i^* + q_j^*} = \frac{a + c_i - 2c_j}{2a - c_i - c_j}, \text{ when } c_j \leq c_i \text{ (i.e. } q_j^* \geq q_i^*), \tag{4}$$

solving (2) in order to c_j,

$$c_j = c - c_i, \text{ with } 0 \leq c_i \leq c. \tag{5}$$

Plugging (5) into (3)

$$C_1 = \begin{cases} \frac{a-3c_i+c}{2a-c} & \text{if } 0 \leq c_i < \frac{c}{2} \\ \frac{a+3c_i-2c}{2a-c} & \text{if } \frac{c}{2} \leq c_i \leq c \end{cases}.$$

The Concentration index is a function of c_i, where,

$$\frac{dC_1}{dc_i} = \begin{cases} \frac{-3}{2a-c} & \text{if } 0 < c_i < \frac{c}{2} \\ \frac{3}{2a-c} & \text{if } \frac{c}{2} < c_i < c \end{cases},$$

hence, it's clear that C_1 reaches the minimum at $c_i = c_j = \frac{c}{2}$ and increases as we move away (in any direction) from that symmetric equilibrium. (*3 points*)

Alternatively, for those who prefer to compute for the Herfindahl index,

$$H = (q_i^*)^2 + \left(q_j^*\right)^2 = \left(\frac{a - 2c_i + c_j}{3b}\right)^2 + \left(\frac{a + c_i - 2c_j}{3b}\right)^2, \tag{6}$$

Plugging (5) into (6)

$$H = \left(\frac{1}{3b}\right)^2 \left[(a + c - 3c_i)^2 + (a - 2c + 3c_i)^2\right].$$

The Herfindahl index is a function of c_i, where,

$$\begin{aligned} \frac{dH}{dc_i} &= 18\left(\frac{1}{3b}\right)^2 [2c_i - c] \text{ and} \\ \frac{d^2H}{dc_i^2} &= 36\left(\frac{1}{3b}\right)^2 > 0. \end{aligned}$$

From the first and second derivatives it's easy to show that the H index function is globally convex and at the point $c_i = \frac{c}{2}$, where $\frac{dH}{dc_i} = 0$, the H index reaches a global minimum. We can conclude from here that *(i)* the concentration index is minimum when firms are symmetric; *(ii)* as firms become more asymmetric (in terms of marginal costs) the H index increases.

The total quantity and price of equilibrium,

$$\begin{aligned} q^* &= q_i^* + q_j^* = \frac{a - 2c_i + c_j}{3b} + \frac{a + c_i - 2c_j}{3b} = \frac{2a - c_i - c_j}{3b} \\ p^* &= \frac{a + c_i + c_j}{3}. \end{aligned}$$

The firm i's equilibrium profit is given by

$$\pi_i^* = \frac{1}{b}\left(\frac{a + c_j - 2c_i}{3}\right)^2,$$

hence, the industry profit is,

$$\pi_{Industry}^* = \pi_i^* + \pi_j^* = \frac{1}{b}\left[\left(\frac{a + c_j - 2c_i}{3}\right)^2 + \left(\frac{a + c_i - 2c_j}{3}\right)^2\right]. \tag{7}$$

Plugging (5) into (7), we get

$$\pi_{Industry}^* = \frac{1}{b}\left[\left(\frac{a + c}{3} - c_i\right)^2 + \left(\frac{a - 2c}{3} + c_i\right)^2\right],$$

where,

$$\frac{d\pi^*_{Industry}}{dc_i} = \frac{2}{b}(2c_i - c) \text{ and}$$
$$\frac{d^2\pi^*_{Industry}}{dc_i^2} = \frac{4}{b} > 0.$$

From the first and second derivatives it's easy to show that the $\pi^*_{Industry}$ function is globally convex and at the point $c_i = \frac{c}{2}$, where $\frac{d\pi^*_{Industry}}{dc_i} = 0$, the industry profit reaches a global minimum. We can conclude from here that *(i)* the industry is less profitable when competing firms are symmetric; *(ii)* as firms become more asymmetric (in terms of marginal costs) the industry yields higher profit levels. (*3 points*)

Part II
Bertrand Competition

(*7 points*)

$$P = 100 - Q$$
$$MgC = 10 = c_1 = c_2$$

a) (*1 point*) Before firm 1's new market strategy,

$$P^* = MgC = 10$$
$$Q = 100 - P \Longleftrightarrow Q^* = 90$$
$$\pi_1^* = \pi_2^* = (P^* - 10)\, q^* = (10 - 10)\, 45 = 0$$

b) Firm 2's problem is,

$$\max_{p_2} \pi_2 = (p_2 - 10)\,\frac{Q}{2} = (p_2 - 10)\,\frac{100 - p_2}{2}$$

$$FOC : \frac{100 - p_2}{2} - \frac{p_2 - 10}{2} = 0 \Longleftrightarrow p_2^* = 55 \text{ and } Q^* = 45. \quad (\textit{1 point})$$
$$SOC : -\frac{1}{2} - \frac{1}{2} = -1 < 0.$$

(*0.5 points*) Firm 2 knows that firm 1 is going to match the price in any case. Thus, the best response from firm 2 is to choose $p_2 = P^{monopoly}$ and split the demand by two, each firm sells $q_1^* = q_2^* = 22.5$ and

$$\pi_1^* = \pi_2^* = (55 - 10)\,\frac{45}{2} = 1012.5.$$

c) (*1.5 points*) Firm 1 has incentive to undercut the price by ε and keep the whole market for itself. Thus, if firm 2 doesn't know that firm 1 undercuts the price, then in equilibrium

$$p_1 = p_2 - \varepsilon = 55 - \varepsilon,$$

where $\varepsilon > 0$ and as small as firm 1 wants, in the limit $\varepsilon \longrightarrow 0$ and so,

$$\begin{aligned} q_1^* &= Q^* = 45, \\ \pi_1^* &= \pi^{monopoly} = 2025 \\ \pi_2^* &= 0. \end{aligned}$$

Alternatively, if you assume that firm 2 knows perfectly firm 1's strategy, then they will come back to the usual Bertrand equilibrium,

$$\begin{aligned} P^* &= MgC = 10 \\ Q &= 100 - P \Longleftrightarrow Q^* = 90 \\ \pi_1^* &= \pi_2^* = (P^* - 10)\, q^* = (10 - 10)\, 45 = 0. \end{aligned}$$

d) (*1 point*)

$$\begin{aligned} \text{Before:} \quad & \pi_1^* = 0 \\ \text{After:} \quad & \pi_1^* = 1012,5. \end{aligned}$$

Therefore, the "aggressive" marketing strategy described in *b)* should be used since firm 1 is better off.

e) (*1 point*)

$$\begin{aligned} \text{Before:} \quad & \pi_2^* = 0 \\ \text{After:} \quad & \pi_2^* = 1012,5. \end{aligned}$$

Therefore, the "aggressive" marketing strategy described in *b)* is also good for firm 2.

f) Computing the consumer surplus variation,

$$\Delta CS = \frac{(100 - 55) \times 45}{2} - \frac{(100 - 10) \times 90}{2} = -\frac{6075}{2} = -3037.5. \quad (\textit{1 point})$$