Industrial Organization

Problem Set #4 Solution

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1 Dynamic Price Competition and Tacit Collusion

Exercise 1.

In period t the monopoly profit is defined by,

$$\pi_t = (P_t - c)\, q_t = (P_t - c)\, \mu^t D(P_t) = \mu^t \pi_0$$

The monopoly price is sustainable if

$$\frac{\pi_0^m}{n}\left(1 + \delta\mu + \delta^2\mu^2 + ...\right) \geq \pi_0^m \tag{1}$$

The left hand side of (1) is the payoff from not deviate; the right hand side is the deviation payoff.

That is,

$$\frac{\pi_0^m}{n}\frac{1}{1-\delta\mu} \geq \pi_0^m \Longleftrightarrow \delta\mu \geq 1 - \frac{1}{n}$$

For a given δ, this condition is more easily satisfied if the market is expanding. The intuition is that the future weights more heavily in such circunstances. If the demand is expanding this means that future profits are higher and so deviate now imply big future losses.

Exercise 2.

a) With no multimarket contact, sustainability of collusion in market 2 iff

$$\pi^M + \delta\pi^M + 0 + 0 + ... \leq \frac{\pi^M}{2}\left(\frac{1}{1-\delta}\right) \Longleftrightarrow \delta \geq \sqrt{\frac{1}{2}}.$$

b) The optimal deviation is to start with market 2 and then deviate on both markets in the following period (a deviation in market 1 triggers punishment in the following period).

Thus, if deviate, the payoff is

$$\pi^M + \delta\pi^M + \frac{\pi^M}{2} + \delta\pi^M = \frac{3}{2}\pi^M + 2\delta\pi^M.$$

The payoff with no deviation is

$$2 \times \frac{\pi^M}{2}\left(\frac{1}{1-\delta}\right).$$

Then, with multimarket contact, sustainability of collusion iff,

$$\begin{aligned} \frac{3}{2}\pi^M + 2\delta\pi^M &\leq 2 \times \frac{\pi^M}{2}\left(\frac{1}{1-\delta}\right) \Longleftrightarrow \frac{1}{2} + \frac{1}{2}\delta - 2\delta^2 \leq 0 \\ \text{Solution is} &: \left(-\infty, -\frac{1}{8}\sqrt{17} + \frac{1}{8}\right] \cup \left[\frac{1}{8}\sqrt{17} + \frac{1}{8}, \infty\right) \end{aligned}$$

However we just care about the positive values, hence,

$$\delta \geq \frac{1}{8} + \sqrt{\frac{17}{64}} \approx 0.640388.$$