Industrial Organization

Problem Set #5 Solution

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Barriers to Entry

Exercise 1.

a) The monopolist's problem with **technology A,**

$$
\begin{aligned}
\max_{q_1} \pi_1^A &= (20 - q_1)\, q_1 - 60 - 2q_1 \\
FOC &: 20 - 2q_1 = 2 \Longleftrightarrow q_1^* = 9 \\
P^* &= 20 - 9 = 11 \\
\pi_1^{A^*} &= 99 - 60 - 18 = 21
\end{aligned}
$$

The monopolist's problem with **technology B,**

$$
\begin{aligned}
\max_{q_1} \pi_1^B &= (20 - q_1)\, q_1 - 10 - 8q_1 \\
FOC &: 20 - 2q_1 = 8 \Longleftrightarrow q_1^* = 6 \\
P^* &= 20 - 6 = 14 \\
\pi_1^{B^*} &= 84 - 10 - 8 \times 6 = 26
\end{aligned}
$$

Since $\pi_1^{B^*} > \pi_1^{A^*}$ a permanent monopolist prefers technology B.

b) Firms compete à la Cournot.
Firm 1's problem is,

$$
\begin{aligned}
\max_{q_1} \pi_1 &= (20 - q_1 - q_2)\, q_1 - c_1 q_1 - F_1 \\
\frac{d\pi_1}{dq_1} &= 0 \Longleftrightarrow 20 - 2q_1 - q_2 - c_1 = 0 \Longleftrightarrow q_1 = \frac{20 - c_1 - q_2}{2},
\end{aligned}
$$

and from firm 2's problem,

$$q_2 = \frac{20 - c_2 - q_1}{2}.$$

In equilibrium,

$$\left\{\begin{array}{l} q_1 = \frac{20-c_1-q_2}{2} \\ q_2 = \frac{20-c_2-q_1}{2} \end{array}\right. \iff \left\{\begin{array}{l} q_1^* = \frac{c_2-2c_1+20}{3} \\ q_2^* = \frac{c_1-2c_2+20}{3} \end{array}\right. .$$

$$\begin{aligned} Q^* &= q_1^* + q_2^* = \frac{40-c_1-c_2}{3} \\ P^* &= \frac{20+c_1+c_2}{3} \\ \pi_1^* &= \left(\frac{20-2c_1+c_2}{3}\right)^2 - F_1 \\ \pi_2^* &= \left(\frac{20-2c_2+c_1}{3}\right)^2 - F_2 \end{aligned}$$

- If both firms with technology A,

$$\begin{aligned} c_1 &= c_2 = 2 \text{ and } F_1 = F_2 = 60, \text{ then the payoffs are} \\ \pi_1^* &= \pi_2^* = \left(\frac{20-4+2}{3}\right)^2 - 60 = -24. \end{aligned}$$

- If both firms with technology B,

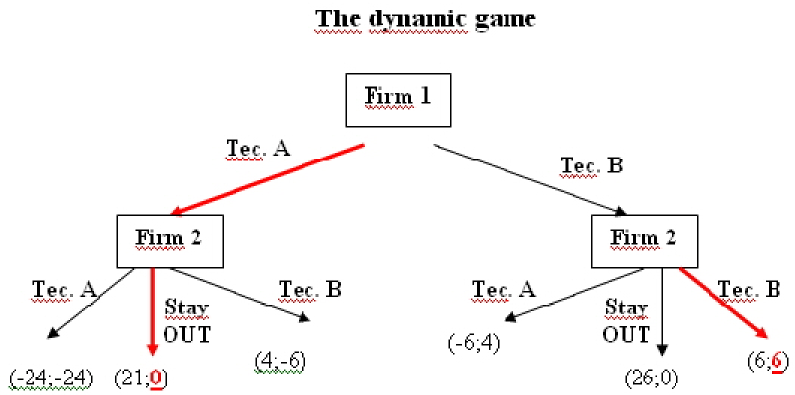
$$\begin{aligned} c_1 &= c_2 = 8 \text{ and } F_1 = F_2 = 10, \text{ then the payoffs are} \\ \pi_1^* &= \pi_2^* = \left(\frac{20-16+8}{3}\right)^2 - 10 = 6 \end{aligned}$$

- If firm 1 with technology A and firm 2 with technology B,

$$\begin{aligned} (c_1, F_1) &= (2, 60)\,;\ (c_2, F_2) = (8, 10)\,, \text{ then the payoffs are} \\ \pi_1^* &= \left(\frac{20-4+8}{3}\right)^2 - 60 = 4 \\ \pi_2^* &= \left(\frac{20-16+2}{3}\right)^2 - 10 = -6 \end{aligned}$$

- If firm 1 with technology B and firm 2 with technology A,

$$\begin{aligned} (c_1, F_1) &= (8, 10)\,;\ (c_2, F_2) = (2, 60)\,, \text{ then the payoffs are} \\ \pi_1^* &= \left(\frac{20-16+2}{3}\right)^2 - 10 = -6 \\ \pi_2^* &= \left(\frac{20-4+8}{3}\right)^2 - 60 = 4 \end{aligned}$$



The equilibrium strategies are, (Tec.A; (Stay OUT, Tec. B))

Firm 1 will choose "Tec. A" forcing firm 2 to choose "Stay Out".

Welfare Analysis

- When only firm 1 is in the market (initial situation) and chooses tec. B, then,

$$W = CS + \pi_1 = \frac{20-14}{2} \times 6 + 6 \times 6 - 10 = 44.$$

- When there's the possibility of a second firm to enter in the market, then firm 1 chooses tec. A and,

$$W = \frac{20-11}{2} \times 9 + 21 = 61.5.$$

Hence, the effect in welfare of the existence of a potential competitor is,

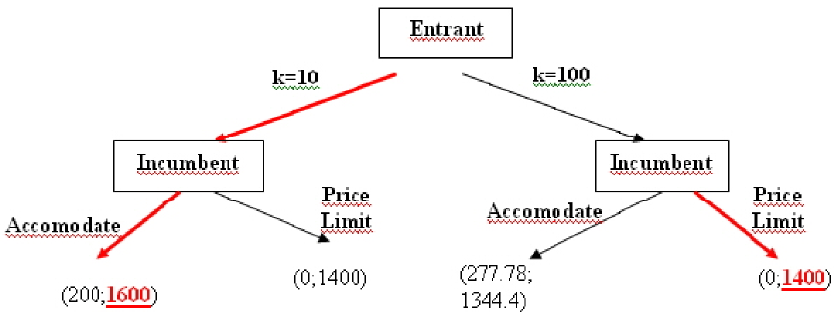
$$\Delta W = 61.5 - 44 = 17.5$$

Exercise 2.
a)

$$\begin{aligned} \max_{q_e} \pi_e &= (100 - q_i - q_e - 30)\, q_e \\ FOC &: 70 - q_i - 2q_e = 0 \iff q_e = \frac{70 - q_i}{2} \end{aligned}$$

And from the incumbent's problem we get,

$$q_i = \frac{90 - q_e}{2}$$



If entrant's capacity, k**, is 100**, and the incumbent accomodates, then,

$$\begin{cases} 2q_e + q_i = 70 \\ q_e + 2q_i = 90 \end{cases} \iff \begin{cases} q_e^* = \frac{50}{3} > k_1 = 10 \\ q_i^* = \frac{110}{3} \end{cases}$$

$$\begin{aligned} Q^* &= q_i^* + q_e^* = \frac{160}{3} \\ P^* &= \frac{140}{3} \\ \pi_i^* &= \left(\frac{140}{3} - 10\right)\frac{110}{3} = \frac{12100}{9} = 1344.4 \\ \pi_e^* &= \left(\frac{140}{3} - 30\right)\frac{50}{3} = \frac{2500}{9} = 277.78 \end{aligned}$$

If the incumbent uses the limit pricing strategy, by 2's reaction function

$$q_2 = \frac{70 - q_1}{2},$$

in order to have $q_e = 0$, then $q_i = 70 \Longrightarrow P^* = 100 - 70 = 30$ and $\pi_i = (30 - 10) \times 70 = 1400$. Hence, with the limit pricing strategy, firm 1 has $\pi_i = 1400 > 1344.4$ and $\pi_e = 0$.

If entrant's capacity, k**, is equal to 10**, and the incumbent accomodates, then,

$$\begin{aligned} q_e &= 10 \\ q_i &= \frac{90 - 10}{2} = 40 \\ Q &= 40 + 10 = 50 \\ P &= 100 - 50 = 50 \\ \pi_i &= (50 - 10) \times 40 = 1600 > 1400. \\ \pi_e &= (50 - 30) \times 10 = 200. \end{aligned}$$

If the incumbent uses the limit pricing strategy, then

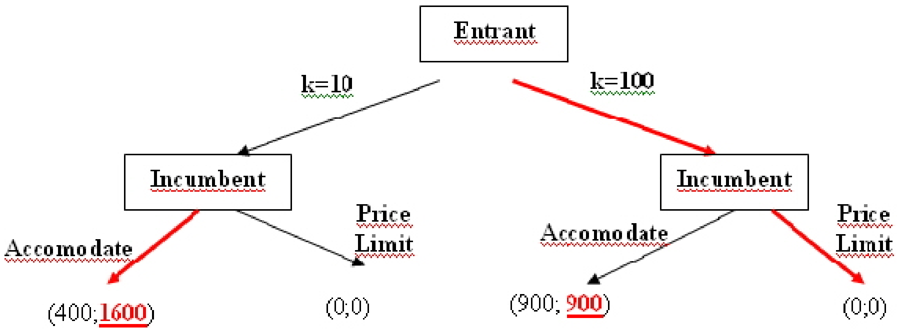
$$\begin{aligned} \pi_e &= 0 \\ \pi_i &= 1400. \end{aligned}$$

Conclusion: Firm 2 installs capacity $k = 10$, since if $k = 100$ then the incumbent would prefer the limit pricing strategy leading to $\pi_e = 0$.

b) Once the entrant is operating in the market, both firms will compete à la Cournot, the outcome is

$$\begin{aligned} q_e^* &= 10 \\ q_i^* &= \frac{90-10}{2} = 40 \\ Q^* &= 40 + 10 = 50 \\ P^* &= 100 - 50 = 50 \\ \pi_i^* &= (50-10) \times 40 = 1600 \\ \pi_e^* &= (50-30) \times 10 = 200. \end{aligned}$$

c) Now, best-response functions are symmetric.



If the **entrant chooses a capacity of** 100 and the incumbent acomodates the entry, thus

$$\begin{cases} q_e = \frac{90-q_i}{2} \\ q_i = \frac{90-q_e}{2} \end{cases} \Longleftrightarrow \begin{cases} q_e = 30 \\ q_i = 30 \end{cases}$$

$$\begin{aligned} Q &= 60 \\ P &= 40 \\ \pi_e &= \pi_i = (40-10) \times 30 = 900. \end{aligned}$$

If the **entrant's capacity is** 10 and the incumbent accomodates the entry, thus

$$\begin{aligned} q_e &= 10 \\ q_i &= 40 \\ Q &= 50 \\ P &= 50 \\ \pi_e &= (50-10)\times 10 = 400 < 900 \\ \pi_i &= (50-10)\times 40 = 1600. \end{aligned}$$

Independently of the capacity chosen by the entrant, if the incumbent plays the **limit pricing strategy**, thus,

$$\begin{aligned} q_e &= 0 \\ q_i &= 90 = Q,\ P = 10 \text{ and } \pi_i = \pi_e = 0. \end{aligned}$$

Conclusion: If the firm that is considering entry has access to a technology equal to the technology of the firm already installed, then the potential entrant chooses a capacity of 100. This happens because if the incumbent plays the limit pricing strategy the outcome is zero profit for both firms. Hence, the best strategy for the incumbent is always to accomodate the entry independently of the capacity chosen by the entrant. The entrant chooses 100 of capacity and the new equilibrium is

$$\begin{aligned} (q_i^*, q_e^*) &= (30, 30) \\ Q^* &= 60,\ P^* = 40,\ \pi_e^* = \pi_i^* = 900. \end{aligned}$$